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MODELING ERROR PROPAGATION IN ENGINEERING CALCULATIONS INVOLVING SINGLE VARIABLE FUNCTIONS

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ABSTRACT

This Research employs both Taylor theorem of expansion and Binomial Coefficient expansion to develop a mathematical model for error which propagates in Engineering Calculations that involve single variable function. Verification of the model was done through computer simulation by comparing the result of the model with the actual difference between the result given by the function in question, when computed with 'error variable' and the same function when computed with 'error free variable'. The output from the model proposed, gave good results for error propagation that were not significantly different from that obtained from the actual difference between the results of the function computed with 'error variable' and 'error free variable'.

***Key words**: Development, Mathematical model, Error propagation, engineering calculation, single variable functions*

1. INTRODUCTION:

More often than not, Engineering calculations involve mathematical functions; this is because, Real World engineering Systems, processes and phenomena are represented using Mathematical functions. According to **[1] , [2] and [3] ,** Mathematical functions are characterized either by single variable as in g = f (x) or many variables as in g = f (x, y,………z). Most time, engineers resort to representing real live situations using single variable functions because many variables functions are either too difficult to model or too difficult to handle. Thus the applications of single variable functions in engineering problems are just too numerous to mention **[4]** . In any single variable functions, there are at least three sources of error **[1] and [5]** , they include:

(i) Error in measuring initial conditions or initial error in the variable which characterize any given mathematical function
(ii) Error in the parameters (constants) in the function
(iii) An incorrect model of the underlying process

It is always very difficult if not impossible to measure accurately the initial condition, this is because it is either the measuring equipments are malfunctioning, human mistake are in introduced or any other factors responsible. Most of the constants or parameters the mathematical function depends upon are usually results of experimental analysis, and if the constants are not discrete, error is bound to set in, in the model **[6]**. If the mathematical function describing an underlying process is not accurate, it shall be very difficult if not impossible to represent real life situation, even when errors from other sources are not present. Although error that propagates in mathematical functions serves as one of factors militating against our ability to obtain exact results, it turns out that this factor plays a major role in our inability to predict far a head some engineering systems, even though the characteristic mathematical functions are highly deterministic. In fact, in large computations, the situation may go worse, such that our final result becomes invalid, if at any point, the results in error happen to serve as input values in our subsequent calculations. Incidentally, more and more decisions in the development of science and Technology are based on large Scale computations and simulations **[4].** Consequently, in order to gain a better understanding in to error propagation in a single variable functions, this study is aimed at developing a mathematical model to simulate, the response of the functions to initial error in the variable which characterizes the function.

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2. MODEL DEVELOPMENT:

Now, we consider a single variable function $f(x)$ which is continuous and differentiable within the x-range of interest, full Taylor expansion of the function about the point x = a is given by:

$$f(x) = f(a) + (x-a)f^{/}(a) + \frac{(x-a)^2}{2!} f^{//}(a) + ... + \frac{(x-a)^{n-1}}{(n-1)!} f^{n-1}(a) + R_n(x)$$

Where $R_n(x)$, the remainder takes the form:

$$R_n(x)) = \frac{(x-a)^n}{n!} f^{(n)}(\xi)$$

and ξ lies in the range $[a, x]$. Suppose we choose ξ such that $\xi \to a$, we have:

$$R_n(x)) = \frac{(x-a)^n}{n!} f^{(n)}(a)$$

Therefore,

$$f(x) = f(a) + (x-a)f^{/}(a) + \frac{(x-a)^2}{2!} f^{//}(a) + ... + \frac{(x-a)^{n-1}}{(n-1)!} f^{n-1}(a) + \frac{(x-a)^n}{n!} f^{(n)}(a)$$

Putting this in a compact form, we have:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n \tag{i}$$

We consider a small error '$\pm \delta x$' in the value of variable 'x' and we seek to investigate analytically, how this error will propagate in the function $f(x)$. By analogy, it can be seen that:

$$f(x \pm \sigma x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x \pm \delta x - a)^n = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}\left((x-a) + (\pm \delta x)\right)^n \tag{ii}$$

If we expand, introduce Binomial coefficient expansion, and neglect terms containing at least error of second order (i.e. ($\pm$ δx) is assumed to be extremely small in magnitude, thus we have:

$$f(x \pm \delta x) = f(a) + (x-a)f^{/}(a) + (\pm \delta x)f^{/}(a) + (x-a)^2 \frac{f^2(a)}{2!} + 2(x-a)(\pm \delta x)\frac{f^2(a)}{2!}$$
$$+ (x-a)^3 \frac{f^3(a)}{3!} + 3(x-a)^2(\pm \delta x)\frac{f^3(a)}{3!} + ...$$

Now, nth term can be generalized as:

$$\left[(x-a) + (\pm \delta x)\right]^n \frac{f^n(a)}{n!} \approx (x-a)^n \frac{f^n(a)}{n!} + n(x-a)^{n-1}(\pm \delta x)\frac{f^n(a)}{n!}$$

Therefore,

$$f(x \pm \delta x) \approx \sum_{n=0}^{\infty}\left[\left(\frac{f^n(a)}{n!}(x-a)^n\right) + \left(n(x-a)^{n-1}(\pm \delta x)\frac{f^n(a)}{n!}\right)\right]$$

or

$$f(x \pm \delta x) \approx \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n + \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}.n.(x-a)^{n-1}(\pm \delta x) \tag{iii}$$

If we define ‘ $E(x)$ ’ as the propagated error in the $f(x)$ by small Error ‘ $\pm\delta x$ ’ in ‘ x ’, we have:

$$f(x \pm \delta x) \approx f(x) + E(x) \tag{iv}$$

Comparing (i), (iii) and (iv), we see that:

$$\boxed{E(x) = \sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}.n.(x-a)^{n-1}(\pm\delta x)} \tag{v}$$

Stationary Behaviour of propagated Error $E(x)$:

From (v), Let $E(x) \approx \frac{f^{0}(a)}{0!}.0.(\pm\delta x) + \frac{f^{/}(a)}{1!}.1.(\pm\delta x) + \frac{f^{2}(a)}{2!}.2.(x-a)(\pm\delta x)$

Putting $x-a$ to be Δx , and $\pm\delta x$ as δx we have:

$$E(x) \approx f^{1}(a)(\delta x) + f^{2}(a)\Delta x(\delta x)$$

But $f^{/}(a) = 0$ (Stationary Point)

Therefore, $E(x) \approx f^{2}(a)\Delta x(\delta x)$ (vi)

Maximum Condition:

For propagated error $E(x)$ to be maximum, we require expression (vi) to be negative, or $f^{2}(a) < 0$.

Minimum Condition:

For propagated error $E(x)$ to be minimum, we require expression (vi) to be positive, or $f^{2}(a) > 0$.

Condition for Stationary point of Inflection:

For propagated error $E(x)$ to be increasing as well as decreasing at a given stationary point, we require $f^{2}(a) = 0$

Performance Evaluation of the Models:

We’re now ready to test the performance of the model, but we need real numbers to use in the equations, presented here are one set of numbers (and the ones used to develop the results in the next section). You may choose any other set or relevant numbers you prefer.

n = 1 to 32,

x = 2 radians;

Function = f = cos(x);

δx = 0.005;

a = 0.

3. RESULTS AND DISCUSSIONS:

From table 1.0, when n=0, the actual difference between $f(x \pm \delta x)$ and f(x) that is [$f(x \pm \delta x)$ -f(x)] =0.0020921 while proposed model E(x) = 0.0 subsequently the difference between [$f(x \pm \delta x)$ -f(x)] and E(x) = 0.0020921. When n= 1, [$f(x \pm \delta x)$ -f(x)]=0.011175, E(x) = 0.0021034 and their difference = 0.0090712.When n= 2, [$f(x \pm \delta x)$ - f(x)]=0.0069904, E(x) = 0.011175 and their difference = -0.0060332. When n= 3, [$f(x \pm \delta x)$ -f(x)] =0.00093542, E(x) = 0.0069686 and their difference = -0.0060332. When n= 4, [$f(x \pm \partial x)$ -f(x)] =0.0023301, E(x) = 0.00092058 and their difference = 0.0014096. When n= 5,[$f(x \pm \delta x)$ -f(x)]=0.0035412, E(x) = 0.0023229 and their difference = 0.0012183. When n= 6, [$f(x \pm \delta x)$ -f(x)]=0.0033552, E(x) = 0.0035325 and their difference = -0.00017729. When n= 7, [$f(x \pm \delta x)$ -f(x)] =0.0032399, E(x) = 0.0033455 and their difference = -0.00010565. When n= 8, [$f(x \pm \delta x)$ - f(x)]=0.0032531, E(x) = 0.0032303 and their difference = 2.2832 e-5. When n= 9, [$f(x \pm \delta x)$ -f(x)]=0.0032595, E(x) = 0.0032437and their difference = 1.5884e-5. This continues until when n=16, where the values of [$f(x \pm \delta x)$ -f(x)], E(x) and their difference stabilize (up to n=32) and equal to 0.0032587, 0.0032493 and 9.4872 e-6 respectively. In a nut shell, there is no significant difference between the results given by the model developed and the actual difference between the functions.

Table- 1: Comparison of Results given by the developed model and the actual differences between the functions

n	f(x)	f(x+δx)	f(x+δx) - f(x)	E(x)s	[f(x+δx) - f(x)] - E(x)
0	-0.9093	0.90721	0.0020921	0	0.0020921
1	-0.077004	0.065829	0.011175	0.0021034	0.0090712
2	1.7416	1.7486	0.0069904	0.011175	-0.0041851
3	1.1867	1.1877	0.00093542	0.0069686	-0.0060332
4	0.58053	0.58286	0.0023301	0.00092058	0.0014096
5	0.6915	0.69504	0.0035412	0.0023229	0.0012183
6	0.77233	0.77568	0.0033552	0.0035325	-0.00017729
7	0.76176	0.765	0.0032399	0.0033455	-0.00010565
8	0.75599	0.75924	0.0032531	0.0032303	0.000022832
9	0.75657	0.75983	0.0032595	0.0032437	0.000015884
10	0.75683	0.76009	0.003259	0.0032501	0.000008894
11	0.75681	0.76007	0.0032587	0.0032495	9.2546E-06
12	0.7568	0.76006	0.0032587	0.0032492	9.5052E-06
13	0.7568	0.76006	0.0032587	0.0032493	9.4932E-06
14	0.7568	0.76006	0.0032587	0.0032493	9.4868E-06
15	0.7568	0.76006	0.0032587	0.0032493	9.4871E-06
16	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
17	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
18	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
19	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
20	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
21	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
22	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
23	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
24	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
25	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
26	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
27	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
28	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
29	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
30	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
31	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06
32	0.7568	0.76006	0.0032587	0.0032493	9.4872E-06

4. CONCLUSION:

Considering the absolute values of the differences between [$f(x \pm \delta x)$ -f(x)] and E(x) which have the highest value of 0.0090712 and lowest value of 9.4872 e-6, thus, it can be seen that the results given by the developed model is in good agreement with the results given by the actual difference between the functions f(x) and $f(x \pm \delta x)$, and as such the model can be said to be mathematically satisfactory.

5. REFERENCES:

[1] Fasogbon, S. K(2009) "Development of a Mathematical Model for Error Propagation in Engineering Calculations involving many variable functions" African Journal of Mathematics and Computer Science Research. Vol. 2(3), pp 039-041.

[2] Bajpai,A.C, Calus,I.M, and Fairlyey, J. A (1973) "Mathematics for Engineers and Scientists"Vol.1, published by John wiley and sons limited, New York.

[3] Weinstein, Eric W. "Error propagation" from mathworld- Awolfram web resource. Retrieved on the 30th July, 2009 from
http://mathworld.wolfram.com/ErrorPropagation.html

[4] Harry Ku (1996). Notes on the use of propagation of error formulas, Journal of Reseach of National Bureau of standards-C. Engineering and Instrumentation, Vol. 70C, No. 4, pp.263-273

[5] Kostelich, E.J., & Armbruster, D. (1996).Introductory Differential Equations: from Linearity to Chaos (preliminary ed-n). Published by Edward Arnold, London

[6] Appendix 2: "Propagation of uncertainty" retrieved on 30th July, 2009 from:

http://www.physics.uc.edu/~bortner/labs/Physics%20Lab%20web%20site/Appendix%202/Appendix%202%20Error%20Propagation%20htm.htm

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