

MINUS LEAP AND SQUARE LEAP INDICES
 AND THEIR POLYNOMIALS OF SOME SPECIAL GRAPHS

V. R. KULLI

Department of Mathematics,
 Gulbarga University, Gulbarga, 585106, India.

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ABSTRACT

We propose the minus leap index and square leap index of a graph. In this paper, we compute the minus leap and square leap indices and their polynomials of wheel, gear, helm, flower and sunflower graphs.

Keywords: minus leap index, square leap index, wheel.

Mathematics Subject Classification: 05C07, 05C12, 05C76.

1. INTRODUCTION

We consider only finite, simple connected graphs. Let G be a graph with a vertex set $V(G)$ and an edge set $E(G)$. The distance between two vertices u and v of a graph G is the number of edges in a shortest path connecting u and v ; and it is denoted by $d(u, v)$. For a positive integer k and a vertex v in G , the open neighborhood of v is defined as $N_k(v/G) = \{u \in V(G) : d(u, v) = k\}$. The k -distance degree $d_k(v)$ of a vertex v in G is the number of k neighbors of v in G , see [1].

In [1], the first and second leap Zagreb indices were introduced and defined as

$$LM_1(G) = \sum_{u \in V(G)} d_2^2(u) \quad LM_2(G) = \sum_{uv \in E(G)} d_2(u)d_2(v).$$

Recently, some novel variants of leap indices were introduced and studied such as leap hyper-Zagreb indices, [2], sum connectivity leap index and geometric-arithmetic leap index [3], F-leap indices [4], augmented leap index [5].

In [6], Albertson proposed the irregularity index (called as minus index in [7]), and defined as

$$M_i(G) = \sum_{uv \in E(G)} |d(u) - d(v)|.$$

Recently, the square ve -degree index [8] was introduced and defined as

$$Q_{ve}(G) = \sum_{uv \in E(G)} [d_{ve}(u) - d_{ve}(v)]^2.$$

Very recently, some square indices were proposed and studied such as square reverse index [9] and square Revan index [10].

We now propose the minus leap index and square leap index of a graph G as follows:

The minus leap index of a graph G is defined as

$$ML(G) = \sum_{uv \in E(G)} |d_2(u) - d_2(v)|. \quad (1)$$

Corresponding Author: V. R. Kulli

Department of Mathematics, Gulbarga University, Gulbarga, 585106, India.

The square leap index of G is defined as

$$QL(G) = \sum_{uv \in E(G)} [d_2(u) - d_2(v)]^2. \quad (2)$$

Considering the minus leap and square leap indices, we define the minus leap polynomial and square leap polynomial of G as

$$ML(G, x) = \sum_{uv \in E(G)} x^{|d_G(u) - d_G(v)|} \quad (3)$$

$$QL(G, x) = \sum_{uv \in E(G)} x^{[d_G(u) - d_G(v)]^2}. \quad (4)$$

In this paper, exact expressions for the minus leap and square leap indices and their polynomials of some special graphs. For special graphs see [11].

2. WHEELS

The wheel W_n is the join of C_n and K_1 . Clearly, $|V(W_n)| = n+1$ and $E(W_n) = 2n$. The vertex K_1 is called apex and the vertices of C_n are called rim vertices. A graph W_n is shown in Figure 1. Throughout this paper, we consider a wheel W_n with $n+1$ vertices.

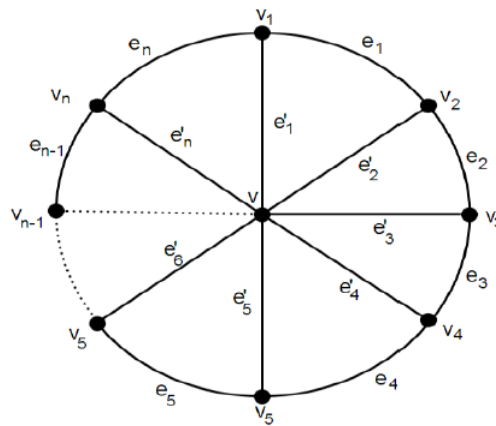


Figure-1: Wheel W_n

Lemma 1: Let W_n be a wheel with $2n$ edges, $n \geq 3$. Then W_n has two types of the 2-distance degree of edges as given below:

$$\begin{aligned} E_1 &= \{uv \in E(W_n) \mid d_2(u) = 0, d_2(v) = n-3\}, & |E_1| &= n. \\ E_2 &= \{uv \in E(W_n) \mid d_2(u) = d_2(v) = n-3\}, & |E_2| &= n. \end{aligned}$$

Theorem 2: Let W_n be a wheel with $n+1$ vertices and $2n$ edges, $n \geq 3$. Then

$$(a) \quad ML(W_n) = n(n-3) \quad (b) \quad QL(W_n) = n(n-3)^2.$$

Proof:

(a) From equation (1) and by Lemma 1, we obtain

$$\begin{aligned} ML(W_n) &= \sum_{uv \in E(W_n)} |d_2(u) - d_2(v)| \\ &= n|0 - (n-3)| + n|(n-3) - (n-3)| = n(n-3). \end{aligned}$$

(b) From equation (2) and by Lemma 1, we obtain

$$\begin{aligned} QL(W_n) &= \sum_{uv \in E(W_n)} [d_2(u) - d_2(v)]^2 \\ &= n[0 - (n-3)]^2 + n[(n-3) - (n-3)]^2 = n(n-3)^2. \end{aligned}$$

Theorem 3: Let W_n be a wheel with $2n+1$ vertices and $2n$ edges. Then

$$(a) \quad ML(W_n, x) = nx^{n-3} + nx^0.$$

$$(b) \quad QL(W_n, x) = nx^{(n-3)^2} + nx^0.$$

Proof:

(a) From equation (3) and by Lemma 1, we have

$$\begin{aligned} ML(W_n, x) &= \sum_{uv \in E(W_n)} x^{|d_2(u) - d_2(v)|} \\ &= nx^{[0 - (n-3)]} + nx^{[(n-3) - (n-3)]} = nx^{(n-3)} + nx^0. \end{aligned}$$

(b) From equation (4) and by Lemma 1, we have

$$\begin{aligned} QL(W_n, x) &= \sum_{uv \in E(G)} x^{\lceil d_2(u) - d_2(v) \rceil^2} \\ &= nx^{[0 - (n-3)]^2} + nx^{[(n-3) - (n-3)]^2} = nx^{(n-3)^2} + nx^0. \end{aligned}$$

3. GEAR GRAPHS

The gear graph G_n is a graph obtained from wheel W_n by adding a vertex between each pair of adjacent rim vertices. Clearly, $|V(G_n)| = 2n+1$ and $|E(G_n)| = 3n$. A gear graph G_n is presented in Figure 2.

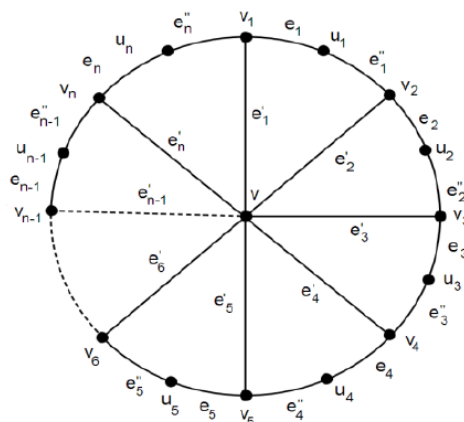


Figure-2: Gear graph G_n

Lemma 4: Let G_n be a gear graph with $3n$ edges, $n \geq 3$. Then G_n has two types of the 2-distance degree of edges as follows:

$$\begin{aligned} E_1 &= \{uv \in E(G_n) \mid d_2(u) = n, d_2(v) = n-1\}, & |E_1| &= n. \\ E_2 &= \{uv \in E(G_n) \mid d_2(u) = 3, d_2(v) = n-1\}, & |E_2| &= 2n. \end{aligned}$$

Theorem 5: Let G_n be a gear graph G_n with $2n$ edges, $n \geq 4$. Then

$$(a) \quad ML(G_n) = 2n^2 - 7n. \quad (b) \quad QL(G_n) = 2n^3 - 16n^2 + 33n.$$

Proof:

(a) By using Lemma 4 and equation (1), we have

$$\begin{aligned} ML(G_n) &= \sum_{uv \in E(G_n)} |d_2(u) - d_2(v)| \\ &= n|n - (n-1)| + 2n|3 - (n-1)| = 2n^2 - 7n. \end{aligned}$$

(b) By using Lemma 4 and equation (2), we deduce

$$\begin{aligned} QL(G_n) &= \sum_{uv \in E(G_n)} [d_2(u) - d_2(v)]^2 \\ &= n[n - (n-1)]^2 + 2n[3 - (n-1)]^2 = 2n^2 - 16n^2 + 33n. \end{aligned}$$

Theorem 6: Let G_n be a gear graph with $3n$ edges. $n \geq 4$ Then

$$(a) \quad ML(G_n, x) = nx + 2nx^{n-4}.$$

$$(b) \quad QL(G_n, x) = nx + 2nx^{(n-4)^2}.$$

Proof:

(a) From equation (3) and by Lemma 4, we obtain

$$\begin{aligned} ML(G_n, x) &= \sum_{uv \in E(G_n)} x^{|d_2(u) - d_2(v)|} \\ &= nx^{|n-n+1|} + 2nx^{|3-n+1|} = nx^1 + 2nx^{n-4}, \text{ since } n \geq 4. \end{aligned}$$

(b) From equation (4) and by Lemma 4, we have

$$\begin{aligned} QL(G_n, x) &= \sum_{uv \in E(G_n)} x^{\lceil d_2(u) - d_2(v) \rceil^2} \\ &= nx^{\lceil n-n+1 \rceil^2} + 2nx^{\lceil 3-n+1 \rceil^2} = nx + 2nx^{(n-4)^2}, \text{ since } n \geq 4. \end{aligned}$$

4. HELM GRAPHS

The helm graph, denoted by H_n , is a graph obtained from W_n by attaching an end edge to each rim vertex. Clearly, $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. A helm graph H_n is shown in Figure 3.

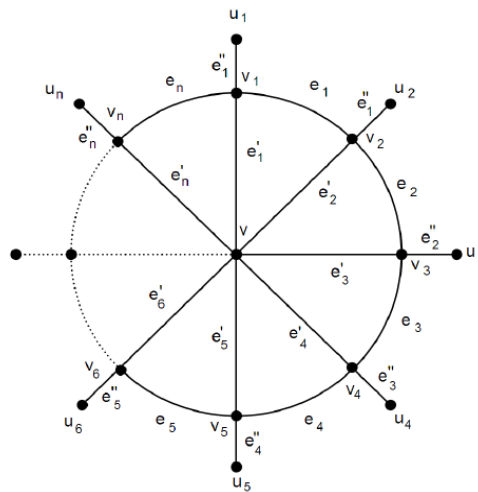


Figure-3: Helm graph H_n

Lemma 7: Let H_n be a helm graph with $3n$ edges, $n \geq 3$. Then H_n has 3 types of the 2-distance degree of edges as follows:

$$\begin{aligned} E_1 &= \{uv \in E(H_n) \mid d_2(u) = n, d_2(v) = n-1\}, & |E_1| &= n. \\ E_2 &= \{uv \in E(H_n) \mid d_2(u) = 3, d_2(v) = n-1\}, & |E_2| &= n. \\ E_3 &= \{uv \in E(H_n) \mid d_2(u) = d_2(v) = n-1\}, & |E_3| &= n. \end{aligned}$$

Theorem 8: Let H_n be a helm graph with $3n$ edges, $n \geq 4$. Then

$$(a) \quad ML(H_n) = n^2 - 3n. \quad (b) \quad QL(H_n) = n^3 - 8n^2 + 17n.$$

Proof:

(a) By using Lemma 7 and equation (1), we deduce

$$\begin{aligned} ML(H_n) &= \sum_{uv \in E(H_n)} |d_2(u) - d_2(v)| \\ &= n|n - (n-1)| + n|3 - (n-1)| + n|(n-1) - (n-1)| = n^2 - 3n. \end{aligned}$$

(b) By using equation (2) and Lemma 7, we derive

$$\begin{aligned} QL(H_n) &= \sum_{uv \in E(H_n)} [d_2(u) - d_2(v)]^2 \\ &= n[n - (n-1)]^2 + n[3 - (n-1)]^2 + n[(n-1) - (n-1)]^2 = n^3 - 8n^2 + 17n. \end{aligned}$$

Theorem 9: Let H_n be a helm graph with $3n$ edges, $n \geq 4$. Then

$$\begin{aligned} (a) \quad ML(H_n, x) &= nx^1 + nx^{n-4} + nx^0. \\ (b) \quad QL(H_n, x) &= nx^1 + nx^{(n-4)^2} + nx^0. \end{aligned}$$

Proof:

(a) From equation (3) and by Lemma 7, we have

$$\begin{aligned} ML(H_n, x) &= \sum_{uv \in E(H_n)} x^{|d_2(u) - d_2(v)|} \\ &= nx^{|n - (n-1)|} + nx^{|3 - (n-1)|} + nx^{|(n-1) - (n-1)|} = nx^1 + nx^{n-4} + nx^0. \end{aligned}$$

(b) From equation (4) and by Lemma 7, we obtain

$$\begin{aligned} QL(H_n, x) &= \sum_{uv \in E(H_n)} x^{\lceil d_2(u) - d_2(v) \rceil^2} \\ &= nx^{\lceil n - (n-1) \rceil^2} + nx^{\lceil 3 - (n-1) \rceil^2} + nx^{\lceil (n-1) - (n-1) \rceil^2} = nx^1 + nx^{(n-4)^2} + nx^0. \end{aligned}$$

5. FLOWER GRAPHS

The flower graph Fl_n is a graph obtained from a helm graph H_n by joining each endvertex to the apex of H_n . Clearly, $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$. A graph Fl_n is presented in Figure 4.

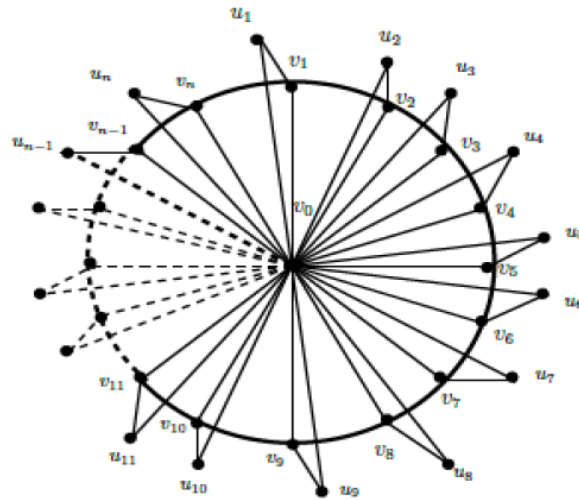


Figure-4: Flower graph Fl_n

Lemma 10: Let Fl_n be a flower graph with $4n$ edges, $n \geq 3$. Then Fl_n has 4 types of the 2-distance degree of edges as given below:

$$\begin{aligned} E_1 &= \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n-5\}, & |E_1| &= n. \\ E_2 &= \{uv \in E(Fl_n) \mid d_2(u) = 0, d_2(v) = n-2\}, & |E_2| &= n. \\ E_3 &= \{uv \in E(Fl_n) \mid d_2(u) = n-5, d_2(v) = n-2\}, & |E_3| &= n. \\ E_4 &= \{uv \in E(Fl_n) \mid d_2(u) = d_2(v) = n-5\}, & |E_4| &= n. \end{aligned}$$

Theorem 11: Let Fl_n be a flower graph with $4n$ edges, $n \geq 3$. Then

$$(a) \quad ML(Fl_n) = 2n^2 - 4n. \quad (b) \quad QL(Fl_n) = 2n^3 - 14n^2 + 38n.$$

Proof:

(a) By equation (1) and by Lemma 10, we have

$$\begin{aligned} ML(Fl_n) &= \sum_{uv \in E(Fl_n)} |d_2(u) - d_2(v)| \\ &= n|0 - (n-5)| + n|0 - (n-2)| + n|(n-5) - (n-2)| + n|(n-5) - (n-5)| = 2n^2 - 4n. \end{aligned}$$

(b) By using equation (2) and Lemma 10, we obtain

$$\begin{aligned} QL(Fl_n) &= \sum_{uv \in E(Fl_n)} [d_2(u) - d_2(v)]^2 \\ &= n[0 - (n-5)]^2 + n[0 - (n-2)]^2 + n[(n-5) - (n-2)]^2 + n[(n-5) - (n-5)]^2 \\ &= 2n^3 - 14n^2 + 38n. \end{aligned}$$

Theorem 12: Let Fl_n be a flower graph with $4n$ edges. $n \geq 3$. Then

$$(a) \quad ML(Fl_n, x) = nx^{n-5} + nx^{n-2} + nx^3 + nx^0.$$

$$(b) \quad QL(Fl_n, x) = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^9 + nx^0.$$

Proof:

(a) From equation (3) and by Lemma 10, we deduce

$$\begin{aligned} ML(Fl_n, x) &= \sum_{uv \in E(Fl_n)} x^{|d_2(u) - d_2(v)|} \\ &= nx^{|0 - (n-5)|} + nx^{|0 - (n-2)|} + nx^{|(n-5) - (n-2)|} + nx^{|(n-5) - (n-5)|} \\ &= nx^{n-5} + nx^{n-2} + nx^3 + nx^0. \end{aligned}$$

(b) From equation (4) and by Lemma 10, we obtain

$$\begin{aligned} Q_L(Fl_n, x) &= \sum_{uv \in E(Fl_n)} x^{\lceil d_2(u) - d_2(v) \rceil} \\ &= nx^{[0-(n-5)]^2} + nx^{[0-(n-2)]^2} + nx^{[(n-5)-(n-2)]^2} + nx^{[(n-5)-(n-5)]^2} = nx^{(n-5)^2} + nx^{(n-2)^2} + nx^9 + nx^0. \end{aligned}$$

6. SUNFLOWER GRAPHS

The sunflower graph Sf_n is a graph obtained from the flower graph Fl_n by attaching n end edges to the apex vertex of Fl_n . Then we have $|V(Sf_n)| = 3n+1$ and $|E(Sf_n)| = 5n$. A graph Sf_n is presented in Figure 5.

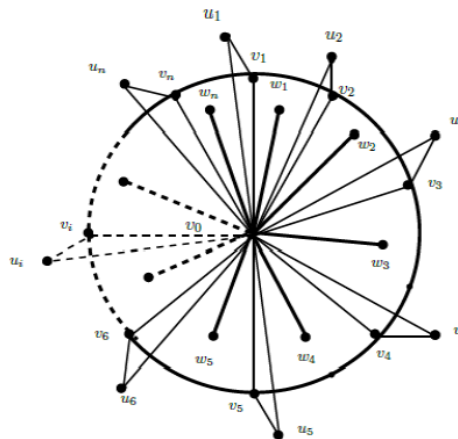


Figure-5: Sunflower graph Sf_n

Lemma 13: Let Sf_n be a sunflower graph with $5n$ edges, $n \geq 3$. Then Sf_n has five types of the 2-distance degree of edges as given below:

$$\begin{array}{ll} E_1 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 4\}, & |E_1| = n. \\ E_2 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 2\}, & |E_2| = n. \\ E_3 = \{uv \in E(Sf_n) \mid d_2(u) = 0, d_2(v) = 3n - 1\}, & |E_3| = n. \\ E_4 = \{uv \in E(Sf_n) \mid d_2(u) = d_2(v) = 3n - 4\}, & |E_4| = n. \\ E_5 = \{uv \in E(Sf_n) \mid d_2(u) = 3n - 4, d_2(v) = 3n - 2\}, & |E_5| = n. \end{array}$$

Theorem 14: Let Sf_n be a sunflower graph with $5n$ edges. Then

(a) $ML(Sf_n) = 9n^2 - 5n$. (b) $QL(Sf_n) = 27n^3 - 42n^2 + 25n$.

Proof:

(a) By equation (1) and by Lemma 13, we deduce

$$\begin{aligned} ML(Sf_n) &= \sum_{uv \in E(Sf_n)} |d_2(u) - d_2(v)| \\ &= n|0 - (3n - 4)| + n|0 - (3n - 2)| + n|0 - (3n - 1)| + n|(3n - 4) - (3n - 4)| \\ &\quad + n|(3n - 4) - (3n - 2)| \\ &= 9n^2 - 5n. \end{aligned}$$

(b) By using equation (2) and Lemma 13, we derive

$$\begin{aligned} QL(Sf_n) &= \sum_{uv \in E(Sf_n)} [d_2(u) - d_2(v)]^2 \\ &= n[0 - (3n-4)]^2 + n[0 - (3n-2)]^2 + n[0 - (3n-1)]^2 + n[(3n-4) - (3n-4)]^2 \\ &\quad + n[(3n-4) - (3n-2)]^2 \\ &= 27n^3 - 42n^2 + 25n. \end{aligned}$$

Theorem 15: Let Sf_n be a sunflower graph with $5n$ edges. Then

- a) $ML(Sf_n, x) = nx^{3n-4} + nx^{3n-2} + nx^{3n-1} + nx^0 + nx^2.$
 b) $QL(Sf_n, x) = nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^0 + nx^4.$

Proof:

(a) From equation (3) and by Lemma 13, we have

$$\begin{aligned} ML(Sf_n, x) &= \sum_{uv \in E(Sf_n)} x^{|d_2(u) - d_2(v)|} \\ &= nx^{|0 - (3n-4)|} + nx^{|0 - (3n-2)|} + nx^{|0 - (3n-1)|} + nx^{|(3n-4) - (3n-4)|} + nx^{|(3n-4) - (3n-2)|} \\ &= nx^{3n-4} + nx^{3n-2} + nx^{3n-1} + nx^0 + nx^2. \end{aligned}$$

(b) From equation (4) and by Lemma 13, we obtain

$$\begin{aligned} QL(Sf_n, x) &= \sum_{uv \in E(Sf_n)} x^{[d_2(u) - d_2(v)]^2} \\ &= nx^{[0 - (3n-4)]^2} + nx^{[0 - (3n-2)]^2} + nx^{[(3n-4) - (3n-4)]^2} + nx^{[(3n-4) - (3n-2)]^2} \\ &= nx^{(3n-4)^2} + nx^{(3n-2)^2} + nx^{(3n-1)^2} + nx^0 + nx^4. \end{aligned}$$

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