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ON GENERALIZED MINIMAL REGULAR SPACES AND GENERALIZED MINIMAL NORMAL SPACES

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ABSTRACT

In this paper the notions of g-minimal regular spaces and g-minimal normal spaces are introduced and studied in topological spaces. A topological space (X, τ) is said to be generalized minimal regular (briefly g-m_i regular) space if for every g-m_i closed set F of X and each point $x \in F^c$ there exists disjoint open sets U and V of X such that $x \in U$ and $F \subset V$. A topological space (X, τ) is said to be generalized minimal normal (briefly g-m_i normal) space if for any pair of disjoint g-m_i closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Some basic properties of such spaces are obtained.

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1. INTRODUCTION AND PRELIMINARIES

N. Levine [2], in 1970 introduced generalized closed (g-closed) sets in topological spaces as a generalization of closed sets. Since then, many concepts related to g-closed sets were defined and investigated. B. M. Munshi [6] has introduced the notions of g-regular spaces and g-normal spaces in topological spaces. Further T. Noiri and V. Popa [9] studied on g-regular spaces and g-normal spaces. Recently minimal open sets and maximal open sets in topological spaces were introduced and characterized by F.Nakaoka and N. Oda ([7], [8]). In section 2, we obtain new characterizations of g-minimal regular spaces whereas in section 3, g-normal spaces are characterized and studied.. The main purpose of this paper is to obtain several preservation theorems of g-m_i regular spaces and g-m_i normal spaces.

Throughout this paper X, Y and Z represent nonempty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, cl (A), int (A) and A^c denote the closure of A, the interior of A and the complement of A in X, respectively. Let us recall the following definitions, which are useful in the sequel.

Definition 1.1: A proper nonempty subset A of a topological space (X, τ) is called
- (i) a minimal open (resp. minimal closed) set [7] if any open (resp. closed) subset of X which is contained in A, is either A or ϕ.
- (ii) a maximal open (resp. maximal closed) set [8] if any open (resp. closed) set which contains A, is either A or X.

Definition 1.2: A subset A of a topological space (X, τ) is called
- (i) a generalized closed[2] (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is an open set in X.
- (i) a generalized minimal closed (briefly g-m_i closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is a minimal open set in X.
- (ii) a generalized maximal open (briefly g-m_a open) set iff A^c is a generalized minimal closed set in X.

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Definition 1.3: A topological space (X, τ) is said to be

(i) g-regular [6] if for each g-closed set F of X and each point $x \in F^c$, there exist disjoint open sets U and V of X such that $x \in U$ and $F \subset V$.

(ii) g-normal[6] if for any pair of disjoint g-closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Definition1.4: A mapping f: $(X, \tau) \to (Y, \sigma)$ is said to be

(i) generalized minimal continuous (briefly g-m_i continuous) map if the inverse image of every minimal closed set in Y is g-minimal closed set in X.

(ii) generalized minimal irresolute (briefly g-m_i irresolute) map if the inverse image of every g- minimal closed set in Y is a g-minimal closed set in X.

Definition 1.5: A map f: $(X, \tau) \to (Y, \sigma)$ is said to be generalized minimal* closed (briefly g- m_i^* closed) map if the image of every g-minimal closed set in X is a g-minimal closed set in Y.

2. GENERALIZED MINIMAL REGULAR SPACES

Definition 2.1: A topological space (X, τ) is said to be generalized minimal regular (briefly g-m_i regular) space if for every g-m_i closed set F of X and each point $x \in F^c$ there exists disjoint open sets U and V of X such that $x \in U$ and $F \subset V$.

Theorem 2.2: Every g-regular space is a g-m_i regular space.

Proof: Let X be a g-regular space and F be a g-m_i closed set in X such that for every $x \in X$, $x \in F^c$. Since every g-m_i closed set is a g-closed set in X, F is a g-closed set in X. But X is g-regular space. Therefore for each g-closed set F in X and each point $x \in F^c$, there exists disjoint open sets U and V in X such that $x \in U$ and $F \subset V$. Thus for every g-m_i closed set F in X and each point $x \in F^c$, there exist disjoint open sets U and V in X such that $x \in U$ and $F \subset V$. Hence X is a g-m_i regular space.

Remark 2.3: Converse of the Theorem 1.2.2 need not be true.

Example 2.4: Let X = {a, b, c, d} with τ ={ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X}.
Open sets: ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X.
Closed sets: ϕ, {a}, {c}, {d}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d},{a, c, d},{b, c, d}, X.
g-closed sets: ϕ, {a}, {c}, {d}, {a, c}, {a, d},{b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X.

Here (X, τ) is a g-m_i regular space but not a g-regular space. Since for a g-closed set F = {d}, F^c = {a, b, c} so that for $b \in F^c$ = {a, b, c}, there do not exist disjoint open sets U and V such that $b \in U$ and $F \subset V$.

Theorem 2.5: In a topological space (X, τ), the following statements are equivalent.

(i) (X, τ) is a g-m_i regular space.

(ii) For each $x \in X$ and for each g-m_a open set U containing x, there exists an open set V such that $x \in V \subset cl (V) \subset U$.

(iii) For each $x \in X$ and for each g-m_i closed set F, such that $x \in F^c$ there exists an open set V such that $x \in V$ and $cl (V) \cap F = \phi$.

Proof:

(i)⇒(ii): Let (X, τ) be a g-m_i regular space and $x \in X$. Let U be any g-m_a open set containing x. Then U^c is a g-m_i closed set such that $x \notin U^c$. Since X is a g-m_i regular space, there exist disjoint open sets V and W of X such that $x \in V$ and $U^c \subset W$.

Now $V \cap W = \phi$ implies $V \subset W^c$ which implies $cl (V) \subset cl (W^c) = W^c$ which implies that $cl (V) \subset W^c$ ……….. (a),

since W^c is a closed set. Again since $U^c \subset W$, $W^c \subset U$…….. (b).

Therefore from (a) and (b), $x \in V \subset cl (V) \subset W^c \subset U$. Thus $x \in V \subset cl (V) \subset U$.

(ii)⇒(iii): For each $x \in X$ let F be any g-m_i closed set in X such that $x \in F^c$. Then F^c is a g-m_a open set containing x. By (ii) there exists an open set V such that $x \in V \subset cl (V) \subset F^c$, which implies $x \notin F$. Therefore $cl (V) \cap F = \phi$.

(iii) ⇒(i): Let $x \in X$ and let F be any g-m_i closed set in X such that $x \in F^c$. By (iii) there exists an open set V such that $x \in V$ and $cl (V) \cap F = \phi$. Since cl (V) is a closed set, $[cl (V)]^c$ is an open set. Now $cl (V) \cap F = \phi$ implies that $F \subset [cl (V)]^c$. Hence for every g-m_i closed set F in X and for each point $x \in F^c$, there exist disjoint open sets V and $[cl (V)]^c$ such that $x \in V$ and $F \subset [cl (V)]^c$. Thus (X,τ) is a g-m_i regular space.

Theorem 2.6: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a bijection, g-m_i irresolute, open map and X is a g-m_i regular space, then Y is a g-m_i regular space.

Proof: Let F be any g-m_i closed set in Y and $y \in F^c$. Since f is a bijective g-m_i irresolute map, there exists $x \in X$ such that $x = f^{-1}(y)$ which implies $f(x) = y$ and $f^{-1}(F)$ is a g-m_i closed set in X. Also $x \in [f^{-1}(F)]^c$. Since X is a g-m_i regular space, by definition for each g-m_i closed set $f^{-1}(F)$ in X such that $x \in [f^{-1}(F)]^c$, there exists disjoint open sets U and V in X such that $x \in U$ and $f^{-1}(F) \subset V$. But f is a bijective open map. Therefore $f(U)$ and $f(V)$ are open sets in Y and $f(U) \cap f(V) = \phi$. Since $U \cap V = \phi$, $f(U \cap V) = \phi$ which implies that $f(U) \cap f(V) = \phi$. Now $x \in U$ implies $f(x) \in f(U)$ which implies that $y \in f(U)$ and $f^{-1}(F) \subset V$ implies $F \subset f(V)$. Therefore for each g-m_i closed set F of Y and for each $y \in F^c$, there exist disjoint open sets $f(U)$ and $f(V)$ in Y such that $y \in f(U)$ and $F \subset f(V)$. Thus Y is a g-m_i regular space.

Theorem 2.7: If f: $(X, \tau) \rightarrow (Y, \sigma)$ is a continuous, g-m_i^* closed, injection and Y is a g-m_i regular space, and then X is a g-m_i regular space.

Proof: Let F be any g-m_i closed set of X and $x \in F^c$. Since f is a continuous g-m_i^* closed map, $f(F)$ is a g-m_i closed set in Y and $f(x) \in [f(F)]^c$. Also since Y is a g-m_i regular space, there exist disjoint open sets U and V such that $f(x) \in U$ and $f(F) \subset V$ which implies that $x \in f^{-1}(U)$ and $F \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Therefore X is a g-$m_i$ regular space.

3. GENERALIZED MINIMAL NORMAL SPACES

Definition 3.1: A topological space (X, τ) is said to be generalized minimal normal (briefly g-m_i normal) space if for any pair of disjoint g-m_i closed sets A and B, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.

Theorem 3.2: Every g-normal space is a g-m_i normal space.

Proof: Let (X, τ) be any g-normal space and let A and B be any pair of disjoint g-m_i closed sets in X. Since every g-m_i closed set is a g-closed set, A and B are g-closed set in X. By hypothesis, for any pair of disjoint g-closed sets A and B there exists disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Therefore for any pair of disjoint g-m_i closed sets A and B, there exists disjoint open sets U and V such that $A \subset U$ and $B \subset V$. Hence (X, τ) is g-m_i normal space.

Remark 3.3: Converse of the Theorem 1.3.2 need not be true.

Example 3.4: Let X = {a, b, c, d} with τ ={ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X}.
Open sets: ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X..
Closed sets: ϕ, {b}, {c}, {a, d}, {b, c}, {a, b, d}, {a, c, d}, X.
g-closed sets: ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, X..
Here (X, τ) is a g-m_i normal space but not a g-normal space. Since
For disjoint g-closed sets {a} and {d} there do not exist disjoint open sets U and V such that $\{a\} \subset U$ and $\{d\} \subset V$.

Theorem 3.5: In a topological space (X, τ), the following statements are equivalent.

(i) (X, τ) is a g-m_i normal space.
(ii) For each g-m_i closed set A and each g-m_a open set U such that $A \subset U$, there exists an open set V such that $A \subset V \subset cl(V) \subset U$.
(iii) For every pair of disjoint g-m_i closed sets A and B of X, there exists an open set V such that $A \subset V$ and $cl(V) \cap B = \phi$.

Proof:
(i) ⇒ (ii): Let (X, τ) be any g-m_i normal space. Let A be a g-m_i closed set A and U be g-m_a open set such that $A \subset U$. Then U^c is a g-m_i closed set in X. Now A and U^c are disjoint g-m_i closed sets. By (i), there exist disjoint open sets V and W such that $A \subset V$ and $U^c \subset W$. Now $V \cap W = \Phi$ implies $V \subset W^c$ which implies $cl(V) \subset cl(W^c) = W^c$ which implies $cl(V) \subset W^c$, since W^c is a closed set. Again, since $U^c \subset W$, $W^c \subset U$. Therefore $A \subset V \subset cl(V) \subset U$.

(ii) ⇒(iii): Let A and B be any pair of disjoint g-m_i closed sets so that $A \cap B = \phi$ then $A \subset B^c$. Since A is a g-m_i closed set and B^c is a g-m_a open set such that $A \subset B^c$, by (ii) there exists an open set V such that $A \subset V \subset cl(V) \subset B^c$ which implies $cl(V) \cap B = \phi$.

(iii) ⇒(i): Let A and B be any pair of disjoint g-m_i closed sets in X.

By (iii), there exists an open set V such that $A \subset V$ and $cl(V) \cap B = \phi$ which implies $A \subset V$ and $B \subset [cl(V)]^c$. Therefore for any pair of disjoint g-m_i closed sets in X, there exists disjoint open sets V and $[cl(V)]^c$ such that $A \subset V$ and $B \subset [cl(V)]^c$. Therefore (X, τ) is a g-m_i normal space.

Theorem 3.6: If f: (X, τ) $\rightarrow$ (Y, σ) is bijection, g-m_i irresolute, open map and X is a g-m_i normal space, then Y is a g-m_i normal space.

Proof: Let A and B be any pair of disjoint g-m_i closed sets in Y. Since f is a g-m_i irresolute map, f^{-1}(A) and f^{-1}(B) are g-m_i closed sets in X and hence $f^{-1}(A) \cap f^{-1}(B) = f^{-1}(A \cap B) = \phi$. But X is a g-$m_i$ normal space, so there exists disjoint open sets U and V such that $f^{-1}(A) \subset U$ and $f^{-1}(B) \subset V$. Since f is open, bijective map, $A \subset f(U)$, $B \subset f(V)$ and $f(U) \cap f(V) = \phi$. Also f (U) and f (V) are open in Y. This shows that Y is a g-m_i normal space.

Theorem 3.7: If f: (X, τ) $\rightarrow$ (Y, σ) is continuous, g-m_i^* closed, injection and Y is a g-m_i normal space, and then X is a g-m_i normal space.

Proof: Let A and B be any pair of disjoint g-m_i closed sets in X. Since f is a g-m_i^* closed map, f(A) and f(B) are g-m_i closed sets in Y and $f(A) \cap f(B) = \phi$. But Y is a g-m_i normal space. So there exist disjoint open sets U and V such that $f(A) \subset U$ and $f(B) \subset V$. Thus we obtain $A \subset f^{-1}(U)$ and $B \subset f^{-1}(V)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Since f is a continuous map, f^{-1}(U) and f^{-1}(V) are open sets in X. This shows that X is a g-m_i normal space.

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