

## The Congruence of k-Curves

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### ABSTRACT

In this paper we introduce the conditions about congruence of K-curves then by using of this conditions stated congruence of curves and surfaces. We prove the conditions about congruence of two curves in  $\mathbf{R}^2$  and  $\mathbf{R}^3$  in different case and discuss congruence of two curves by using the concept of isometry, translation and rotation without using torsion and curvature. And we study half-curves that created the graphs of their conjunctions as follows;



Our purpose of invariant curve is half-curve or segment of curve that less than limited in one side. (see [3,7])

**Keywords:** Congruence Theory, Isometry, Rotation, Translation

### 1. INTRODUCTION:

In elementary differential geometry the condition about congruence of two curves has been considered. And with the extension of congruence of curves discuss of surface congruence stated in the world in the last time. In second reference the theorem of congruence of two curves in  $\mathbf{R}^2$  and  $\mathbf{R}^3$  are express as follows:

$\mathbf{R}$  is the set of real numbers,  $\mathbf{I}$ ,  $\mathbf{I}_1$  and  $\mathbf{I}_2$  are an interval in  $\mathbf{R}$ ,  $\delta \mathbf{I}$  is boundary of interval  $\mathbf{I}$ ,  $\tau$  is torsion of a curve,  $\kappa$  is curvature a curve,  $\mathbf{v}$  is the parallel vector of a curve,  $\mathbf{E}^2$  is Euclidian plane and  $\mathbf{E}^3$  is Euclidian space.

**Definition 1.1:** The two curves  $\alpha$  and  $\beta: \mathbf{I} \rightarrow \mathbf{E}^3$  are congruent if there is an isometry  $F: \mathbf{E}^3 \rightarrow \mathbf{E}^3$  such that  $F(\beta(t)) = \alpha(t)$  for every  $t$  in  $\mathbf{I}$ . (See [1, 3])

**Theorem 1.2:** If  $\alpha$  and  $\beta: \mathbf{I} \rightarrow \mathbf{E}^3$  are two curves such that  $\|\alpha'(t)\| = \|\beta'(t)\| = 1$  for every  $t$  in  $\mathbf{I}$ ,  $\kappa_\alpha = \kappa_\beta$  and  $\tau_\alpha = \pm \tau_\beta$  then  $\alpha$  and  $\beta$  are congruent. (see [3,5])

**Theorem 1.3:** Suppose  $\alpha$  and  $\beta: \mathbf{I} \rightarrow \mathbf{E}^3$  are two curves such that

$$\|\alpha'(t)\| = \|\beta'(t)\| = 1: \text{ for every } t \text{ in } \mathbf{I}, \kappa_\alpha = \kappa_\beta > 0, \tau_\alpha = \pm \tau_\beta$$

and  $\nu_\alpha = \nu_\beta > 0$ , then  $\alpha$  and  $\beta$  are congruent. (see [2,3])

**Theorem 1.4:** Suppose  $\alpha$  and  $\beta: \mathbf{I} \rightarrow \mathbf{E}^3$  are two arbitrary curves.

Let  $\mathbf{E}_1, \mathbf{E}_2$  and  $\mathbf{E}_3$  is a frame field on  $\alpha$  and  $\mathbf{F}_1, \mathbf{F}_2$  and  $\mathbf{F}_3$  is a frame field on  $\beta$ ,

if  $\alpha' \cdot \mathbf{E}_i = \beta' \cdot \mathbf{F}_i$  ( $1 \leq i \leq 3$ ) and  $\mathbf{E}_i' \cdot \mathbf{E}_j = \mathbf{F}_i' \cdot \mathbf{F}_j$  ( $1 \leq i, j \leq 3$ ), then  $\alpha$  and  $\beta$  are congruent. (see [3])

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**Theorem 1.5:** Suppose  $\alpha$  and  $\beta: I \rightarrow E^2$  are two curves such that

$$\|\alpha'(t)\| = \|\beta'(t)\| = 1: \text{ for every } t \text{ in } I.$$

Then  $\alpha$  and  $\beta$  in  $E^2$  are congruent if and only if  $\kappa_\alpha = \pm \kappa_\beta$ . (see [3,4,6])

## 2. THE RESULTS:

**Definition 2.1: (Junction Point)** Suppose  $\alpha: I_1 \rightarrow R^2$  and  $\beta: I_2 \rightarrow R^2$  are two segment curves or half-curves. In this case we say the unique point  $p \in \delta I_1 \cap \delta I_2$  is junction point if  $\alpha(p) = \beta(p)$ .

In simple words we say the points that created by conjunction of two half-curves or segment curves are junction points.

**Definition 2.2: (Broken k-Curve)** A continuous curve is called broken k-curved if it has k corner point and without closed region in its graph.

**Definition 2.3: (Invariant 2-Curve)** If  $\alpha$  and  $\beta$  are two invariant curves such that of their conjunction in junction point that created a broken 1-curve or broken 2-curve, we say it invariant 2-curves and we show it with  $\overline{\alpha\beta}$  notation.

In simple words invariant 2-curves are the same curved as follows;



**Definition 2.4: (k-Curves by n)** Suppose  $\alpha$  is a curve we say  $\alpha$  is k-curve by n if formed by k times invariant curves and we can see less than n times invariant 2- curves in its graph.

**Definition 2.5: (invariant k-curve (k>2))** If  $\alpha_1, \dots, \alpha_k$  are k invariant curves and  $\overline{\alpha_i \alpha_{i+1}}$  ( $i=1, \dots, k-1$ ) are invariant 2-curves such that  $P_i$  is junction point for  $\overline{\alpha_i \alpha_{i+1}}$ , then broken (k-1)-curves  $\overline{\alpha_1 \alpha_2} \dots \overline{\alpha_{k-1} \alpha_k}$  is called an invariant k-curves, or, The  $\alpha$  curve formed by k times invariant curves and its graph has (k-1) times invariant 2-curves such that every two invariant 2-curves with disjoint junction point are common at most in one invariant curve.

**Corollary 2.6:** Every k-curve by n is not an invariant k-curve.

For example the following figure is a 3-curve by 3 but is not an invariant 3-curve.



**Corollary 2.7:** Every invariant k-curve is continuous. It is clear by definition of invariant k-curve.

**Theorem 2.8:** The union of two isometry is an isometry. (see [3,8])

**Theorem 2.9: (Invariant k-Curves Conformity)** Suppose  $\alpha$  and  $\beta$  are two invariant k-curves that  $\alpha$  includes from k curves  $\{\alpha_1, \dots, \alpha_k\}$  and  $\beta$  includes from k-curves  $\{\beta_1, \dots, \beta_k\}$  then  $\alpha$  on  $\beta$  is congruent if ; there is a translation F such that  $F(\alpha) = \beta$  or there is a rotation F such that  $F(\alpha) = \beta$  or there is an isometry F such that  $F(\alpha) = \beta$ .

**Proof:** If the invariant curves of  $\alpha_i$  ( $i=1, 2, 3 \dots k$ ) congruent on  $\beta_i$  by isometry G, then it is enough to put translation F equal G. (By the last theorem the union of two isometry is isometry).

**Theorem 2.10: (khosravi-faryad theorem)** Every two same shapes in  $R^2$  are congruent by at most of two translations and one rotation.

**Proof:** Suppose  $S_1$  and  $S_2$  are two same shapes in  $R^2$  such that are in different places. We will show that  $S_1$  and  $S_2$  are congruent by at most two translations and one rotation. We congruence the centroid of  $S_1$  and  $S_2$  by at most of two translations.

Let  $\theta$  is the angle of between smallest surrounded rectangles  $S_1$  and  $S_2$ .

Now with rotation around in this point to measure of  $\theta$ , the congruence is possible.

In other words every point of surrounded rectangle of  $S_1$  input to point of surrounded rectangle  $S_2$  with rotation to measure of  $\theta$ , especially the region of  $S_1$  input to  $S_2$ .

**Lemma 2.11:** Every two same rectangular parallelepiped with the same centroid in  $\mathbf{R}^3$  are congruent at most by two rotations.

**Proof:** Suppose A and B are two same rectangular parallelepiped in  $\mathbf{R}^3$  with the same centroid.

Suppose  $S_1$  is a perpendicular sheet on four parallel ribs  $a_1, a_2, a_3$  and  $a_4$  from A.

There are four parallel ribs  $b_1, b_2, b_3$  and  $b_4$  in B such that have the same length with  $a_1, a_2, a_3$  and  $a_4$  i.e.  $|a_i| = |b_i|$  ( $i=1, 2, 3, 4$ ).

Let  $S_2$  is a perpendicular sheet on  $b_1, b_2, b_3$  and  $b_4$ , with a rotation to measure angle of between two sheet normal vectors  $S_1$  and  $S_2$  around centroid this two sheets are congruent together.

Now located A and B in one direction i.e.  $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4$  are parallel together.

Now we consider four rectangles of perpendicular ribs on  $a_i$  ( $i=1, 2, 3, 4$ ) from A and perpendicular ribs on  $b_i$  ( $i=1, 2, 3, 4$ ) from B. These rectangles have the same center two by two. A and B are congruent with together by a rotation to measure angle of between these two rectangles around their centroid.

**Theorem 2.12: (khosravi-faryad theorem)** Every two same shapes in  $\mathbf{R}^3$  are congruent by at most three translations and two rotations.

**Proof:** Suppose  $S_1$  and  $S_2$  are the same shapes in  $\mathbf{R}^3$  and locates in different places.

At first by three translation, the centroid of  $S_1$  and  $S_2$  are congruent. Then by last lemma there are at most two rotations such that smallest surrounded rectangular parallelepiped of two shapes  $S_1$  and  $S_2$  are conformed, so  $S_1$  and  $S_2$  will be congruent.

**Conjecture:** Every two same shapes in  $\mathbf{R}^n$  are at most congruent by n translation and (n-1) rotation.

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