



**HALL EFFECTS ON MHD OSCILLATORY FLOW  
OF A MICROPOLAR FLUID OVER A VERTICAL POROUS PLATE**

**K. KASHAIAH\*, Prof. V. DHARMAIAH**

**Department of Mathematics, Osmania University Hyderabad, India.**

*(Received On: 14-05-16; Revised & Accepted On: 12-06-16)*

---

**ABSTRACT**

*We observed the Hall effects on MHD oscillatory flow of a micro polar fluid through a porous medium over a vertical porous plate. The expressions for the velocity field, angular velocity and temperature field are obtained analytically. It is examined that the axial velocity u increases with increasing  $m, Gr$  and  $\beta$  while it decreases with an increase in  $M, Pr$  and  $n$ . Also, it is observed that, the micro-rotation vector  $\omega$  decreases with increasing  $m, Gr$  and  $\beta$ , while it increases with increasing  $M, Pr$  and  $n$*

---

**1.1. INTRODUCTION**

The theory of micro polar fluid can be used to explain the flow behavior of non-Newtonian fluids, such as colloidal fluids, polymeric fluids, animal blood and real fluids with suspensions. This theory, first formulated by Eringen [33] could deal with viscous fluids where the micro constituents are rigid and spherical or randomly oriented. It has got much attention in recent years. An excellent review about micro polar fluids was provided by Ariman *et al.* [7, 8].

The problem of fluid flow and heat transfer past a porous plate has attracted the interest of many investigators in view of its applications in many engineering problems such as oil exploration, geothermal energy extractions and the boundary layer control in aerodynamics (Soundalgekar [87]; Kim [51]; Kim [52]; Rapti [76]). Specifically, Soundalgekar [87] obtained approximate solutions for the two dimensional flow of an incompressible, viscous fluid flow past an infinite porous vertical plate. He found that the difference between the temperature of the plate and the free stream is significant to cause the free convection currents. Gorla and Tornabene [38] have investigated the effects of thermal radiation on mixed convection flow over a vertical plate with non-uniform heat flux boundary conditions. Raptis [76] studied numerically the case of a steady two-dimensional flow of a micro polar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. Kim [52] studied the unsteady free convection flow of a micro polar fluid through a porous medium bounded by an infinite vertical plate.

The effect of the magnetic field on free convection flows is important in liquid metals, electrolytes and ionized gases. Soundalgekar and Takher [88] have studied the effect of MHD force and free convective flow past a semi-infinite plate. Raptis and Kafousias [75] studied the influence of a magnetic field upon the steady free convection flow through a porous medium bounded by an infinite vertical plate with a constant suction velocity and when the plate temperature is also constant. The problem of MHD mixed convection flow of Newtonian fluids over a horizontal plate has been analyzed by Ibrahim and Hady [49]. Hassanien and Hady [43] have investigated the effects of Hydro magnetic free convection and mass transfer flow of a non-Newtonian fluid through a porous medium bounded by an infinite vertical limiting surface with constant suction. Kim and Lee [53] have discussed the MHD oscillatory flow of a micro polar fluid over a vertical porous plate. Singh [86] has studied MHD free convection and mass transfer flow with Hall current, viscous dissipation, joule heating and thermal diffusion. Makinde and Mhone [55] have studied heat transfer to MHD oscillatory flow in a channel filled with porous medium. MHD flow and heat transfer in a viscoelastic liquid over a stretching sheet in the presence of radiation was investigated by Siddheshwar and Mahabaleshwar [85]. Mostafa [59] has studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Reddappa *et al.* [66] have studied the MHD

oscillatory flow of a micro polar fluid through a porous medium over a vertical porous plate. The analytical study of MHD radiation-convection with surface temperature oscillation and secondary flow effects was studied by Beg and Gosh [14].

In this chapter we investigate the Hall effects on MHD oscillatory flow of a micro polar fluid through a porous medium over a vertical porous plate. The expressions for the velocity field, angular velocity and temperature field are obtained analytically. The effects of time  $t$ , Hartmann number  $M$ , Darcy number  $Da$ , Grashof number  $Gr$ , viscosity ratio  $\beta$  and micro-rotation vector  $\omega$  on the velocity field, angular velocity and temperature field are studied in detail.

## 1.2. MATHEMATICAL FORMULATION

We consider the unsteady laminar two-dimensional convection flow of an incompressible micro polar fluid over a semi-infinite vertical porous moving plate in the presence of a magnetic field. It is assumed that the transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible when comparing with applied magnetic field. The Hall Effect is taken into account. It is also assumed here that the pore size of the porous plate is significantly larger than a characteristic microscopic length scale of a micro polar fluid. The  $x^*$ -axis is taken along the planar surface in the upward direction and the  $y^*$ -axis is taken to be normal to it. Due to the semi-infinite plane surface assumption, the flow variables are functions of  $y^*$  and the time  $t^*$  only. Fig. 1.2.1 shows the physical model of the problem.

Under these conditions, the mass, linear momentum, micro-rotation and energy conservation equations can be written as follows:

$$\frac{\partial v^*}{\partial y^*} = 0 \quad (1.2.1)$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = (v + v_r) \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f(T - T_\infty) - \frac{\sigma B_0^2}{\rho(1+m^2)} u^* + 2v_r \frac{\partial w^*}{\partial y^*} \quad (1.2.2)$$

$$\rho j^* \left( \frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} \right) = r \frac{\partial^2 w^*}{\partial y^{*2}} \quad (1.2.3)$$

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha_1 \frac{\partial^2 T}{\partial y^{*2}} \quad (1.2.4)$$

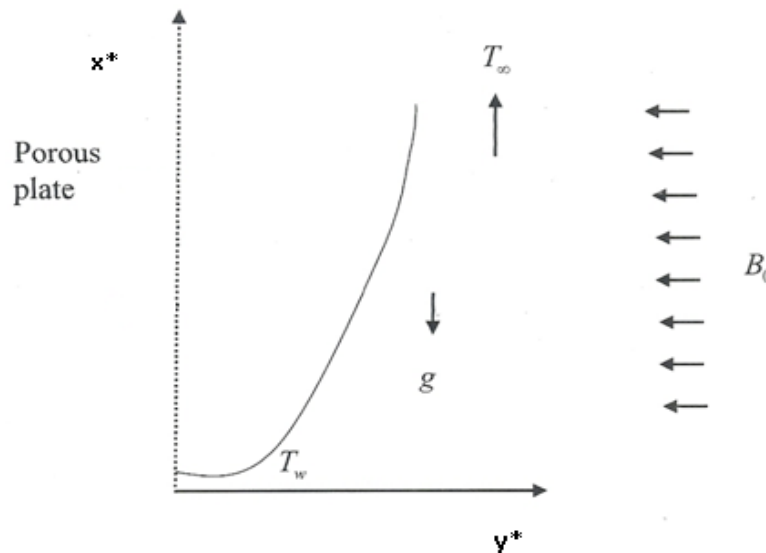


Fig.1.2. 1: The Physical model

where  $u^*, v^*$  are the components of dimensional velocities along  $x^*$  and  $y^*$  directions, respectively,  $\rho$  is the fluid density,  $\nu$  - the kinematic viscosity,  $\nu_r$  - the kinematic rotational viscosity,  $g$  - the acceleration of gravity,  $\beta_f$  - the coefficient of volumetric thermal expansion of the fluid,  $m$  - the Hall parameter,  $\sigma$  - the electrical conductivity of the fluid,  $B_0$  - the magnetic induction,  $j^*$  - the micro-inertia density,  $\omega^*$  - the component of the angular velocity vector normal to the  $x^* y^*$ -plane,  $\gamma$  - the spin gradient viscosity,  $T$  - the temperature and  $\alpha_2$  - the effective fluid thermal diffusivity.

The second term of the RHS of the momentum Eq. (1.2.2) denotes buoyancy effects. The heat due to viscous dissipation is omitted for small velocities in energy equation. It is assumed that the porous plate moves with constant velocity  $u_p^*$  in the longitudinal direction, and the plate temperature ( $T$ ) varies exponentially with time.

Under these assumptions, the appropriate boundary conditions for the velocity and temperature fields are.

$$\begin{aligned} u^* &= u_p^*, T = T_w + \varepsilon(T_w - T_\infty)e^{i\xi^*t^*}, \omega^* = -n \frac{\partial u^*}{\partial y^*} \text{ at } y^* = 0 \\ u^* &\rightarrow 0, T \rightarrow T_\infty, \omega^* \rightarrow 0 \text{ as } y^* \rightarrow \infty \end{aligned} \quad (1.2.5)$$

In the boundary condition for micro-rotation variable  $\omega^*$  that describes its relationship with the surface stress, the parameter  $n$  is a number between 0 and 1 that relates the micro-rotation vector to the shear stress. The value  $n = 0$  corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value  $n = 0.5$  is indicative of weak concentrations, and when  $n = 1$  flows are believed to represent turbulent boundary layers (Rees and Bassom[78]).

Integrating the continuity Eq. (5.2.1) for constant suction, we get

$$v^* = -V_0 \quad (1.2.6)$$

Where  $V_0$  is a scale of suction velocity which has non-zero positive constant.

We now introduce the following dimensionless variables:

$$\begin{aligned} u &= \frac{u^*}{U_0}, u_p = \frac{v^*}{V_0}, y = \frac{V_0 y^*}{\nu}, u_p = \frac{u_p^*}{U_0}, \omega = \frac{\nu}{U_0 V_0} \omega^*, \xi = \frac{\xi^* \nu}{V_0^2}, j = \frac{V_0^2}{\nu^2} j^* \\ t &= \frac{t^* V_0^2}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \text{Pr} = \frac{\nu \rho C_p}{k} = \frac{\nu}{\alpha}, M = \frac{\sigma B_0^2 \nu}{\rho V_0^2} \text{ (magnetic field parameter),} \\ Gr &= \frac{\nu \beta_f g (T_w - T_\infty)}{U_0 V_0^2} \end{aligned} \quad (1.2.7)$$

(Grashoff number),

Where  $U_0$  is a scale of free stream velocity. Further more, the spin-gradient viscosity  $\gamma$  which gives some relationship between the coefficient of viscosity and micro-inertia, is defined as

$$\gamma = \left( \mu + \frac{\Lambda}{2} \right) j^* = \mu j^* \left( 1 + \frac{1}{2} \beta \right), \beta = \frac{\Lambda}{\mu} \quad (1.2.8)$$

Here  $\beta$  denotes the dimensionless viscosity ratio in which  $\Lambda$  is the coefficient of gyro-viscosity (or vortex viscosity). with the help of Equations (5.2.6)-(5.2.8), the governing Equations (1.2.2)-(5.2.4) reduce to the following dimensionless form

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = (1 + \beta) \frac{\partial^2 u}{\partial y^2} Gr \theta - Nu + 2\beta \frac{\partial \omega}{\partial y} \quad (1.2.9)$$

$$\frac{\partial \omega}{\partial t} - \frac{\partial \omega}{\partial y} = \frac{1}{\eta} \frac{\partial^2 \omega}{\partial y^2} \quad (1.2.10)$$

$$\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \theta}{\partial y^2} \quad (1.2.11)$$

where

$$\eta = \frac{\mu j^*}{\gamma} = \frac{2}{2 + \beta} \text{ and } N = M / (1 + m^2).$$

The boundary conditions Eq. (5.2.5) are written as follows:

$$\begin{aligned} u &= u_p, \theta = 1 + \varepsilon e^{i\xi t}, \omega = -n \frac{\partial u}{\partial y} \text{ at } y = 0 \\ u &\rightarrow 0, \theta \rightarrow 0, \omega \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \quad (1.2.12)$$

### 1.3. SOLUTION

In order to reduce the partial differential equations (1.2.9) - (1.2.11) to a system of ordinary differential equations in dimensionless form, we may represent the linear velocity, micro-rotation and temperature as

$$\begin{aligned} u &= u_0 + \varepsilon e^{i\xi t} u_1 + O(\varepsilon^2) \\ \omega &= \omega_0 + \varepsilon e^{i\xi t} \omega_1 + O(\varepsilon^2) \\ \theta &= \theta_0 + \varepsilon e^{i\xi t} \theta_1 + O(\varepsilon^2) \end{aligned} \quad (1.3.1)$$

Substituting Eq. (5.2.13) into Equations (5.2.9) - (5.2.11) and taking only the real parts from the physical point of view, we get the following system of equations for  $(u_0, \omega_0, \theta_0)$  and  $(u_1, \omega_1, \theta_1)$

$$(1 + \beta) \frac{d^2 u_0}{dy^2} + \frac{du_0}{dy} - Nu_0 = Gr\theta_0 - 2\beta \frac{d\omega_0}{dy} \quad (1.3.2)$$

$$(1 + \beta) \frac{d^2 u_1}{dy^2} + \frac{du_1}{dy} (\xi \tan \xi t - N) u_1 = Gr\theta_1 - 2\beta \frac{d\omega_1}{dy} \quad (1.3.3)$$

$$\frac{d^2 \omega_0}{dy^2} + \eta \frac{d\omega_0}{dy} = 0 \quad (1.3.4)$$

$$\frac{d^2 \omega_1}{dy^2} + \eta \frac{d\omega_1}{dy} + n\xi \tan \xi t \omega_1 = 0 \quad (1.3.5)$$

$$\frac{d^2 \theta_0}{dy^2} + \eta \frac{d\theta_0}{dy} = 0 \quad (1.3.6)$$

$$\frac{d^2 \theta_1}{dy^2} + \eta \frac{d\theta_1}{dy} + n\xi \tan \xi t \theta_1 = 0 \quad (1.3.7)$$

However, this expansion of the solution is important only if the reduced equations are ordinary differential equations of independent variable  $y$ . In fact, the solutions to  $u_1, \omega_1, \theta_1$  are time dependent and are not consistent with the assumption. Therefore, we have only considered constant values of  $\xi t$ .

The corresponding boundary conditions can be written as

$$\begin{aligned} u_0 &= Up, u_1 = 0, \omega_0 = -nu_0', \omega_1 = -nu_1', \theta_0 = 1, \theta_1 = 1 \text{ at } y = 0 \\ u_0 &= 0, u_1 = 0, \omega_0 = 0, \omega_1 = 0, \theta_0 = 0, \theta_1 = 0 \text{ as } y \rightarrow \infty \end{aligned} \quad (1.3.8)$$

Solving equations (5.3.2) – (5.3.7) under the boundary condition Eq. (5.3.8) and substituting the solutions into Eq. (5.3.1), we obtain

$$u = a_1 e^{-h_2 y} + a_2 e^{-p r y} + a_3 e^{-\eta y} + \varepsilon (b_1 e^{-h_1 y} + b_2 e^{-h_3 y} + b_e e^{-h_4 y}) e^{i\xi t} \quad (1.3.9)$$

$$\omega = c_1 e^{-\eta y} + \varepsilon (c_2 e^{-h_1 y}) e^{i\xi t} \quad (1.3.10)$$

$$\theta = e^{-Pr y} + \varepsilon (e^{-h_4 y}) e^{i\xi t} \quad (1.3.11)$$

where

$$\begin{aligned} h_1 &= \frac{\eta}{2} \left[ 1 + \sqrt{1 - \frac{4\xi \tan \xi t}{\eta}} \right], h_2 = \frac{1}{2(1 + \beta)} \left[ 1 + \sqrt{1 + 4N(1 + \beta)} \right] \\ h_3 &= \frac{1}{2(1 + \beta)} \left[ 1 + \sqrt{1 - 4N(\xi \tan \xi t - N)(1 + \beta)} \right], h_4 = \frac{Pr}{2} \left[ 1 + \sqrt{\frac{1 - 4\xi \tan \xi t}{Pr}} \right] \\ a_1 &= U_p - a_2 - a_3, a_2 = -\frac{Gr}{(1 + \beta)Pr^2 - Pr - N}, \Phi = \frac{2\beta\eta}{(1 + \beta)\eta^2 - \eta - N}, a_3 = \Phi c_1, \end{aligned}$$

$$\Psi = \frac{2\beta\eta_1}{(1+\beta)h_1^2 - h_1 + (\xi \tan \xi t - N)}, \quad b_1 = \Psi c_2, \quad b_2 = -(b_1 + b_3),$$

$$b_3 = \frac{Gr}{(1+\beta)h_4^2 - h_4 + (\xi \tan \xi t - N)},$$

$$c_1 = \frac{n(h_2 U_1 - h_2 a_2 + \text{Pr } a_2)}{1 + n\Phi(h_2 - \eta)}$$

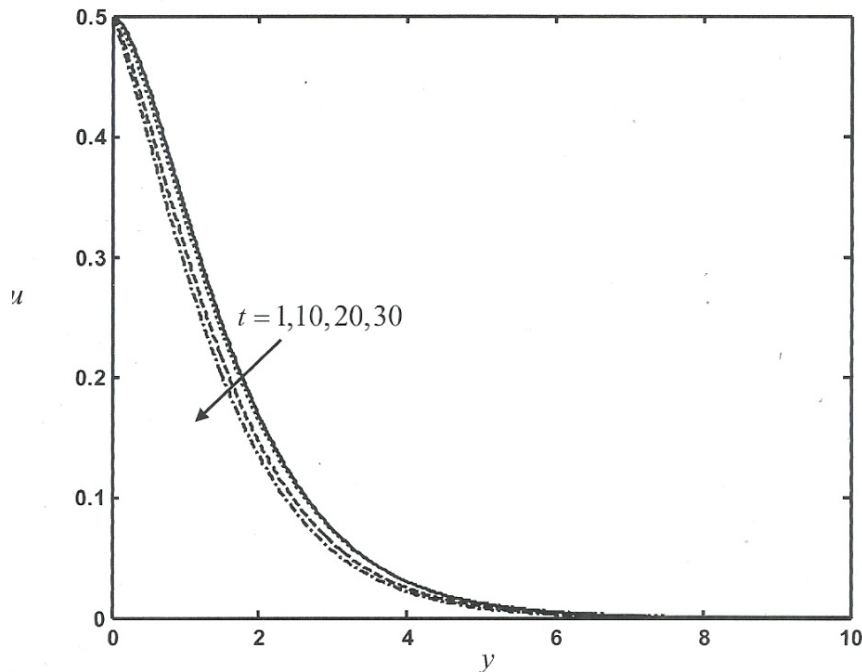
$$c_2 = \frac{nb_3(h_4 - h_3)}{1 + n\psi(h_3 - h_1)}$$

The rate of heat transfer coefficient in terms of Nusselt number  $Nu$  at the wall of the plate is given by

$$Nu = -\frac{\partial \theta}{\partial y} \Big|_{y=0} = \text{Pr} + \varepsilon h_4 e^{i\xi t}$$

when  $m \rightarrow 0$ , our results coincide with those results obtained by Kim and Lee. [53].

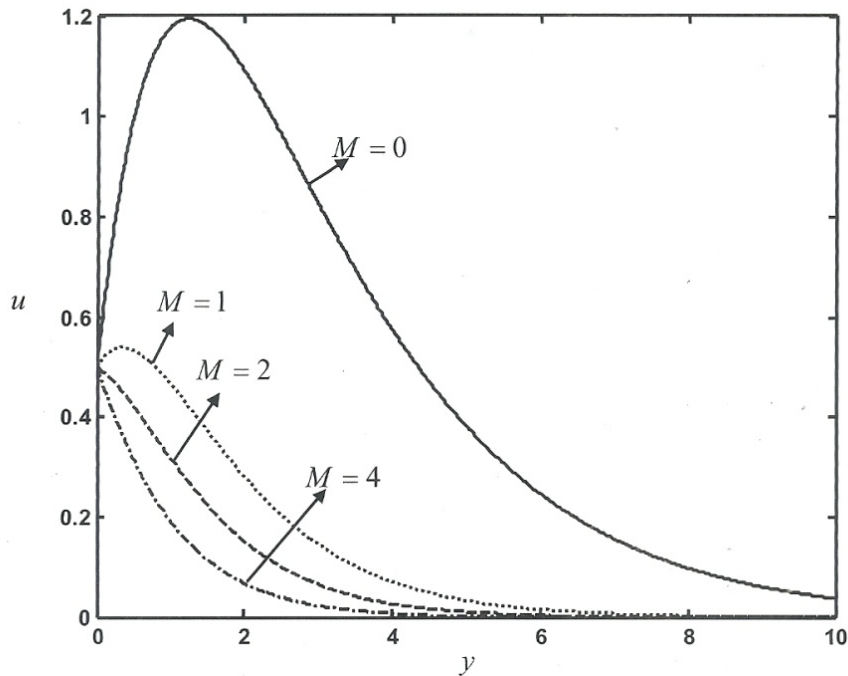
#### 1.4. DISCUSSION OF THE RESULTS



**Fig.1.4.2:** Unsteady velocity profile  $u$  for  $m = 0.2, \text{Pr} = 1, \beta = 1, M = 2, Gr = 2, \varepsilon = 0.1, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$

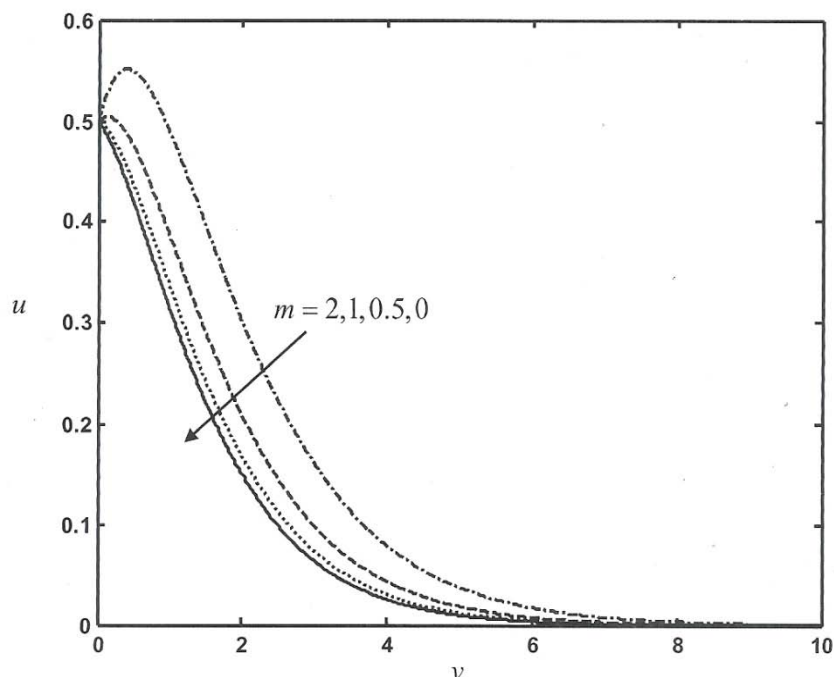
Fig.5.4.2 illustrates the unsteady velocity profiles in the boundary layer for  $\xi = 0.1, \varepsilon = 0.1, n = 0.5, \beta = 1, M = 2, Gr = 2, \text{Pr} = 1$  and  $m = 0.2$ . It is observed that the peak values of velocity are located near the wall of the porous plate. Also, it is found that the behavior of velocity distribution is completely oscillatory.

The variation of the velocity  $u$  with Hartmann number  $M$  for  $m = 0.2, \text{Pr} = 1, \beta = 1, t = 0.1, Gr = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$  is shown in



**Fig.1.4.3:** Effects of Hartmann number  $M$  on velocity profile  $u$  for  $m = 0.2$ ,  $Pr = 1$ ,  $\beta = 1$ ,  $t = 0.1$ ,  $Gr = 2$ ,  $\varepsilon = 1$ ,  $t = 0.1$ ,  $u_p = 0.5$ ,  $n = 0.5$  and  $\xi = 0.1$ .

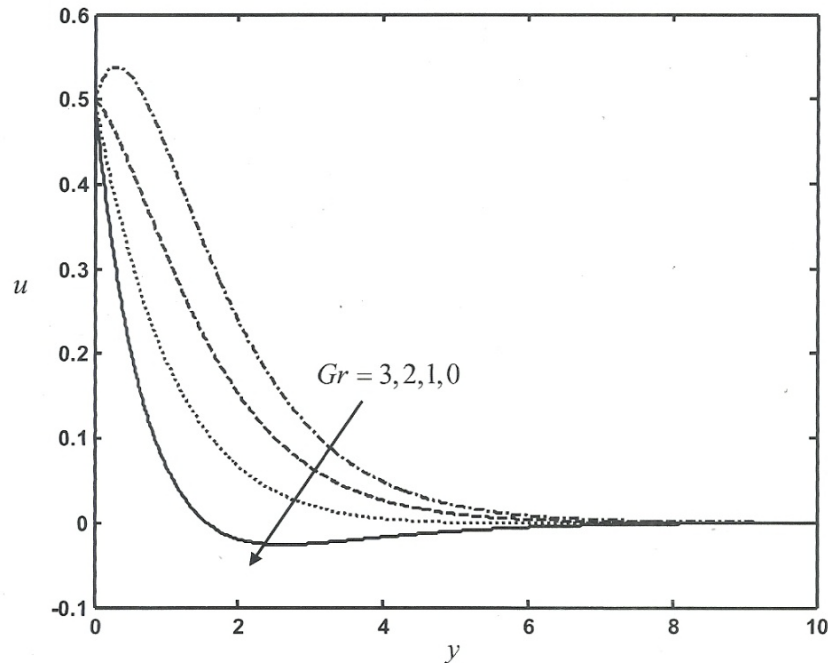
From Fig.1.4.3 it is observed that with increasing Hartmann number  $M$  results in a decreasing velocity distribution across the boundary layer. Further, it is found that, the velocity is more for non-conducting (magnetic) (i.e.  $M \rightarrow 0$ ) micro polar fluid than that of conducting micro polar fluid.



**Fig. 1.4.4:** Effect of Hall parameter  $m$  on velocity profile  $u$  for  $t = 0.1$ ,  $Pr = 1$ ,  $\beta = 1$ ,  $M = 2$ ,  $Gr = 2$ ,  $\varepsilon = 0.01$ ,  $u_p = 0.5$ ,  $n = 0.5$  and  $\xi = 0.1$ .

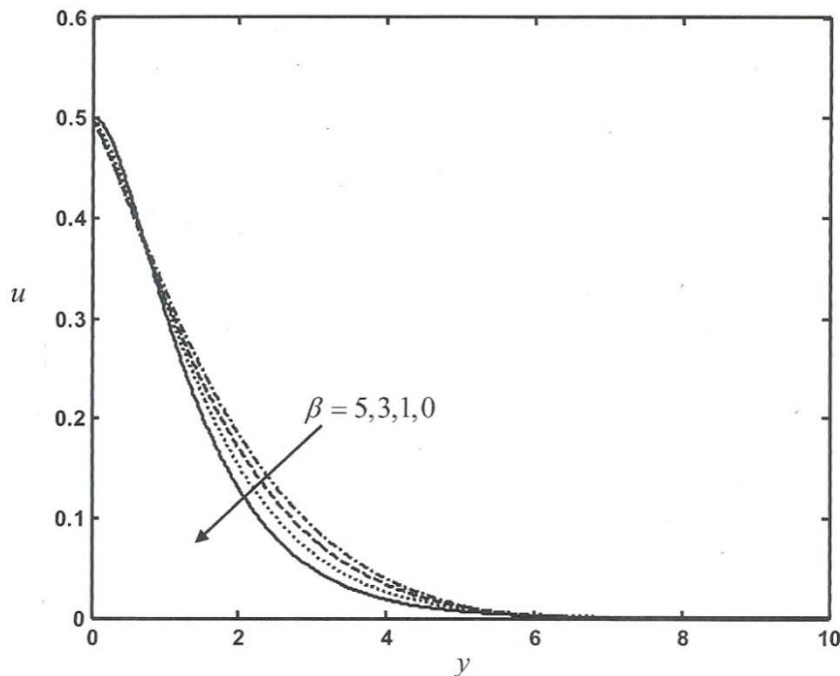
Fig. 1.4.4 depicts the effect of Hall parameter  $m$  on the velocity  $u$  for  $t = 0.1$ ,  $Pr = 1$ ,  $\beta = 1$ ,  $M = 2$ ,  $Gr = 2$ ,  $\varepsilon = 0.01$ ,  $u_p = 0.5$ ,  $n = 0.5$  and  $\xi = 0.1$ . It is found that, the velocity distribution increases with increasing  $m$  across the boundary layer.

The effect of Grashof number  $Gr$  on the velocity  $u$  is shown in



**Fig. 1.4.5:** Effect of Grashof number  $Gr$  on velocity profile  $u$  for  $t = 0.1, Pr = 1, \beta = 1, M = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ .

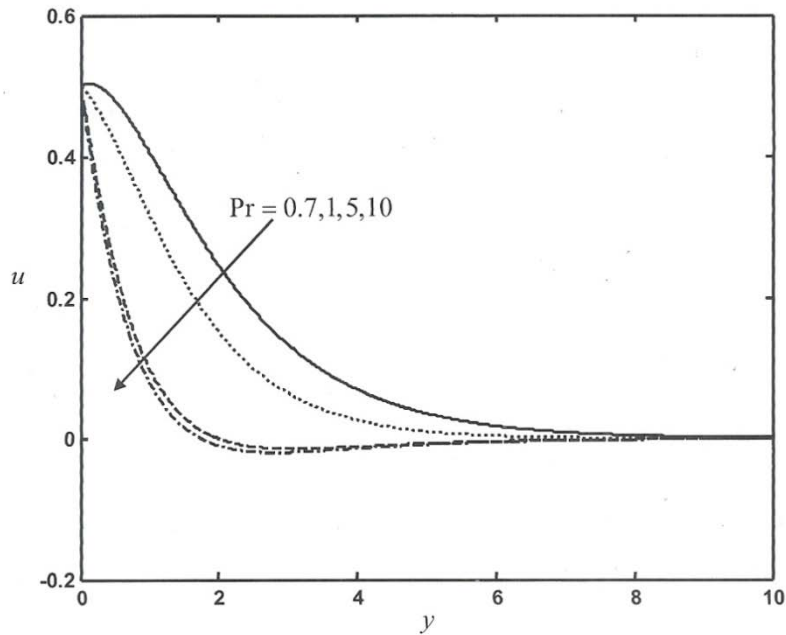
Fig. 1.4.5 for  $t = 0.1, Pr = 1, \beta = 1, M = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . It is found that, an increase in  $Gr$  leads to an increase in velocity due to the enhancement in buoyancy force.



**Fig. 1.4.6:** Effect of micro polar parameter  $\beta$  on velocity profile  $u$  for  $t = 0.1, Pr = 1, Gr = 2, m = 0.2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ .

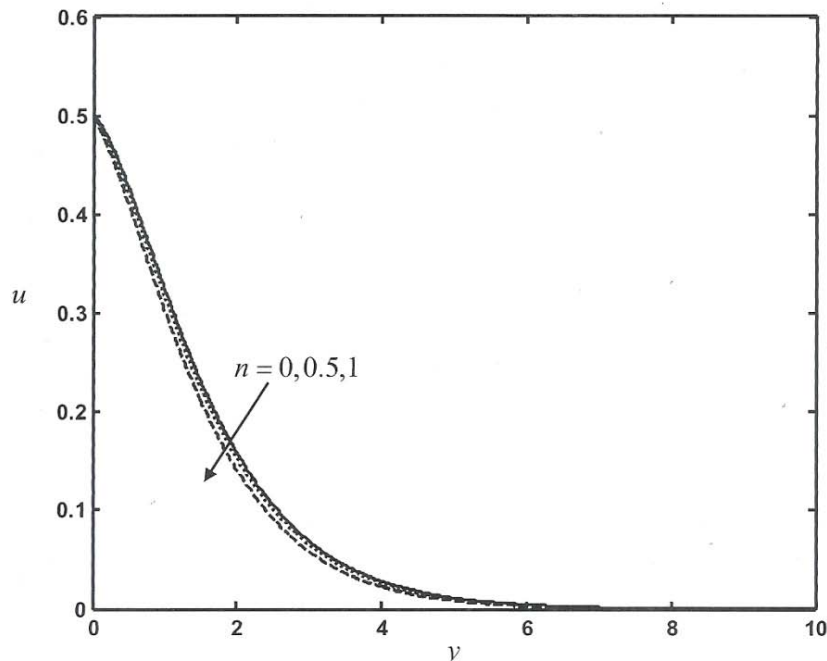
Fig. 1.4.6 illustrates the velocity  $u$  against spanwise coordinate  $y$  for different values of velocity ratio  $\beta$  with  $t = 0.1, Pr = 1, Gr = 2, M = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . The results show that the velocity gradient near the porous plate increases as  $\beta$  increases. Also the velocity distribution across the boundary layer is Higher for Newtonian fluid ( $\beta = 0$ ) for the same flow conditions and fluid properties, as compared with a micropolar fluid, except for near the wall of the porous plate.

Effect of Prandtl number  $Pr$  on velocity profile  $u$  for  $t = 0.1, \beta = 1, Gr = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$  is shown in



**Fig. 1.4.7:** Effect of Prandtl number  $Pr$  on velocity profile  $u$  for  $t = 0.1, \beta = 1, Gr = 2, m = 0.2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ .

Fig.1.4.7 it is found that, the velocity  $u$  decreases with increasing  $Pr$ .

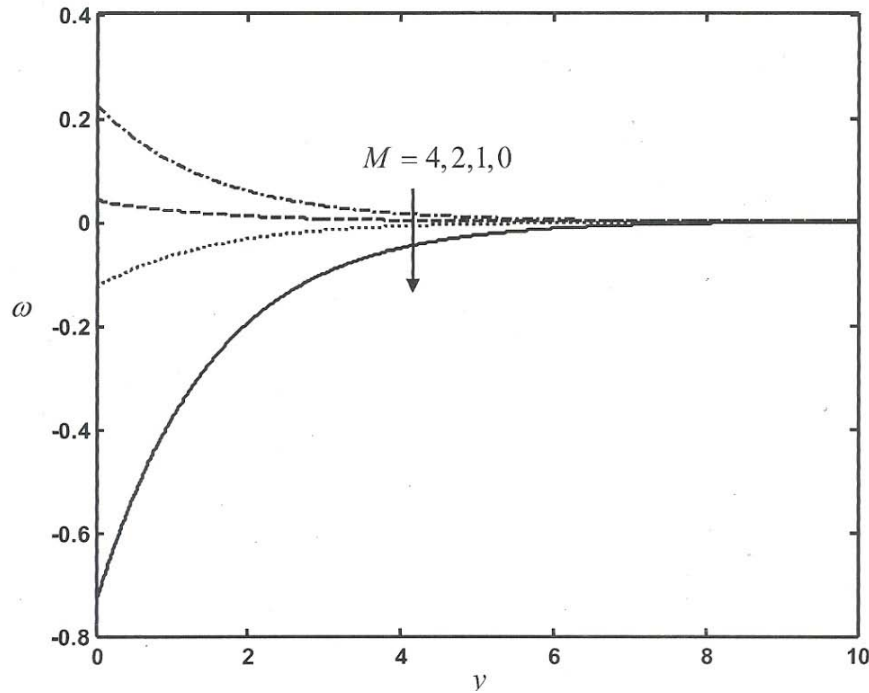


**Fig. 1.4.8:** Effect of micro-gyration parameter  $n$  on velocity profile  $u$  for  $t = 0.1, Pr = 1, Gr = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5, M = 2$  and  $\xi = 0.1$ .

Fig.5.4.8 shows the effect of micro-gyration parameter  $n$  on the velocity for  $t = 0.1, Pr = 1, Gr = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . It is observed that increasing the values of  $n$  results in a decreasing velocity within the boundary layer, which eventually approaches to the relevant free stream velocity at the edge of boundary layer.

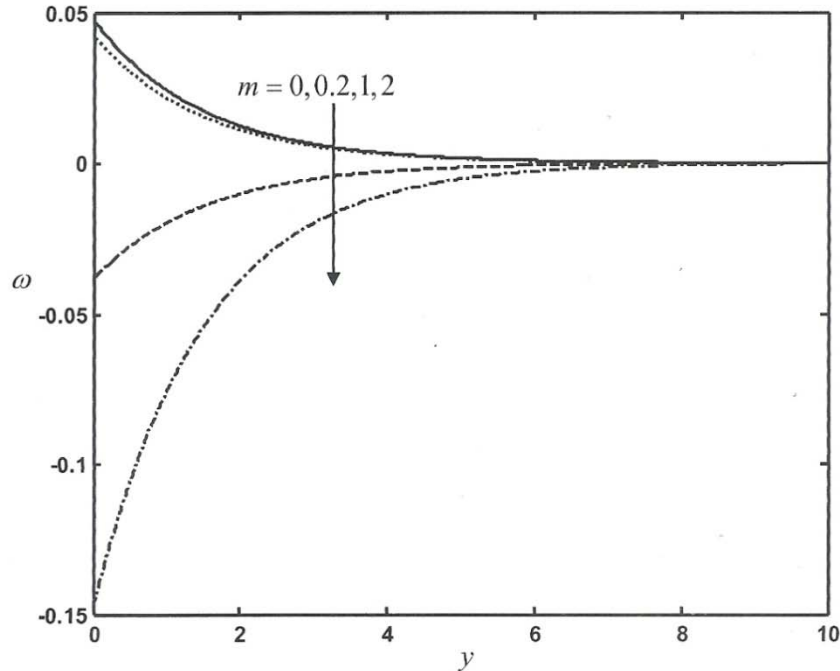


The variation of micro-rotational velocity  $\omega$  with  $M$  for  $m = 0.2, Pr = 1, \beta = 1, t = 0.1, Gr = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$  is depicted in



**Fig. 1.4.9:** Effect of Hartmann number  $M$  on micro-rotational velocity for  $m = 0.2, Pr = 2, \beta = 1, t = 0.1, Gr = 2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ .

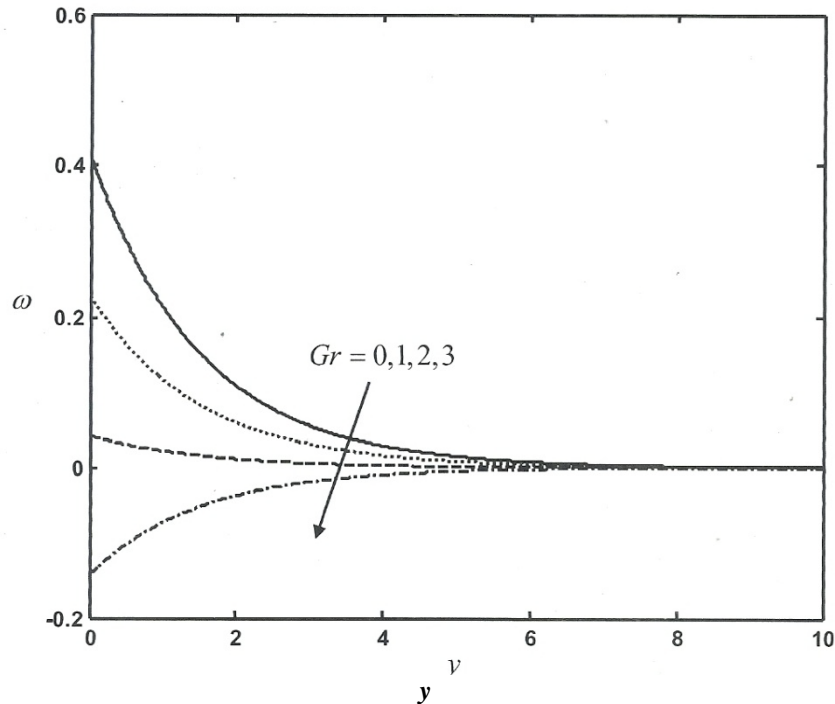
From Fig. 1.4.9 it is observe that the micro-rotational velocity  $\omega$  increases with increasing  $M$ .



**Fig. 1.4.10:** Effects of Hall parameter  $m$  on micro-rotational velocity for  $t = 0.1, Pr = 1, \beta = 1, M = 2, Gr = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$

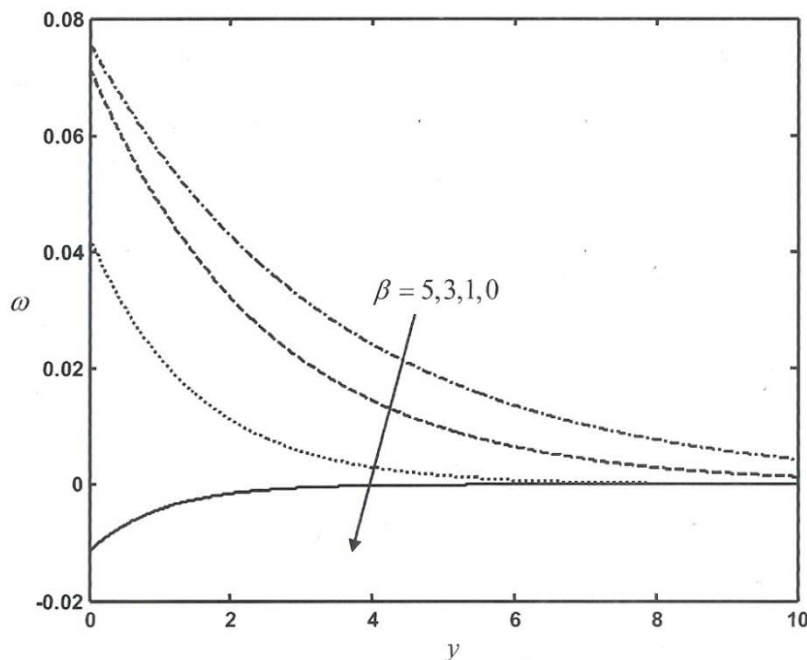
Fig. 1.4.10 depicts the variation of micro-rotational velocity  $\omega$  with  $m$  for  $t = 0.1, Pr = 1, \beta = 1, M = 2, Gr = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . It is found that, an increase in  $m$  results in a decreasing micro-rotational velocity  $\omega$  across the boundary layer.

The micro-rotational velocity  $\omega$  against span wise co-ordinate  $y$  for different values of Grashof number  $Gr$  for  $t = 0.1, Pr = 1, \beta = 1, m = 0.2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$  is shown in



**Fig. 1.4.11:** Effects of Grashof number  $Gr$  on micro-rotational velocity for  $t = 0.1, Pr = 1, \beta = 1, M = 0.2, \varepsilon = 0.01, u_p = 0, n = 0.5, M = 2$  and  $\xi = 0.1$

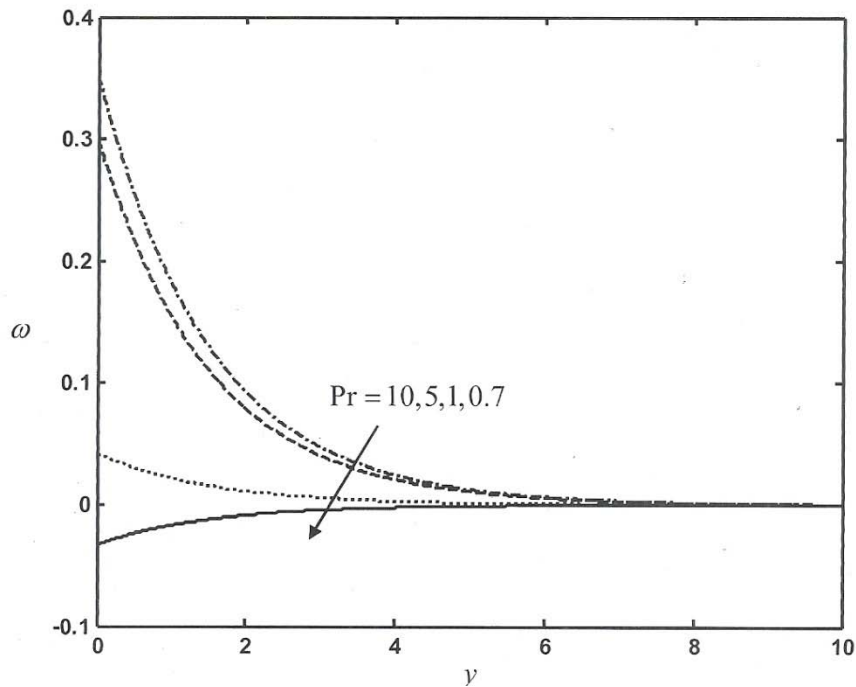
From Fig 1.4.11 it is observe that, micro-rotational velocity  $\omega$  decreases with an increase in  $Gr$ .



**Fig. 1.4.12:** Effects of micropolar parameter  $\beta$  on micro-rotational velocity for  $t = 0.1, Pr = 1, \beta = 1, M = 0.2, Gr = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$

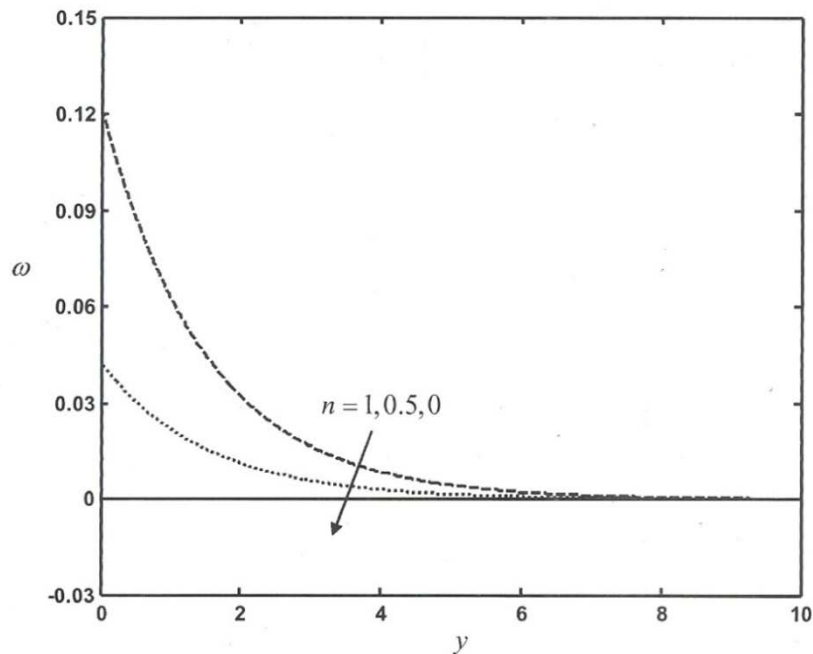
Fig. 5.4.12 shows the effect of viscosity  $\beta$  on the micro-rotational velocity  $\omega$  for  $t = 0.1, Pr = 1, Gr = 2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . It is observed that, near the porous plate micro-rotational velocity  $\omega$  decreases as the viscosity ratio  $\beta$  increases.

The effect of Prandtl number  $Pr$  on micro-rotational velocity for  $t = 0.1, \beta = 1, Gr = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$  is depicted in



**Fig. 1.4.13:** Effects of Prandtl number  $Pr$  on micro-rotational velocity for  $t = 0.1, \beta = 1, Gr = 2, M = 2, m = 0.2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$

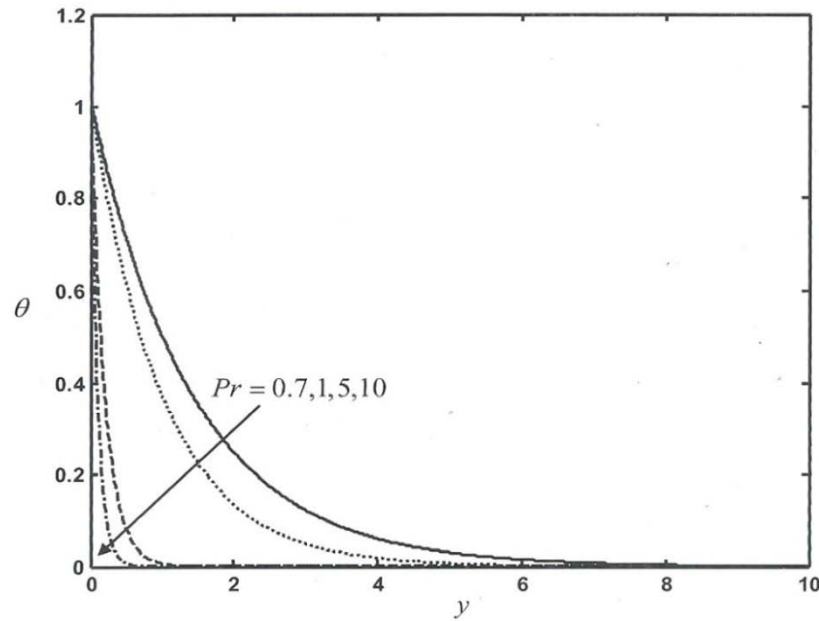
From Fig. 1.4.13 it is found that the micro-rotational velocity  $\omega$  increases with increasing  $Pr$ .



**Fig. 1.4.14:** Effects of micro-gyration parameter  $n$  on micro-rotational velocity for  $Gr = 2, t = 0.1, M = 2, Pr = 1, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$

Fig. 1.4.14 depicts the effects of micro gyration parameter non micro-rotational velocity for  $t = 0.1, Pr = 1, Gr = 2, m = 0.2, M = 2, \varepsilon = 0.01, u_p = 0.5, n = 0.5$  and  $\xi = 0.1$ . It is found that, the micro-rotational velocity  $\omega$  increases with increasing  $n$ .

The variation of temperature  $\theta$  with Prandtl number  $Pr$  for  $\varepsilon = 0.01, \xi = 0.1, t = 0.1$  is shown in



**Fig. 1.4.15:** Temperature distribution with different values of Prandtl number  $Pr$  for  $\xi = 0.1, \varepsilon = 0.01$  and  $t = 0.1$ . From Fig. 1.4.15. It is found that, an increase in the Prandtl number  $Pr$  results in a decreasing in the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of  $Pr$  are equivalent to increasing the thermal conductivities, and therefore heat is able to spread away from the heated surface more rapidly than for higher values of  $Pr$ . Hence the boundary layer is thicker and the rate of heat transfer is reduced. The gradients have been reduced.

**Table – 1.4.1:** Effect of  $Pr$  on Nusselt number  $Nu$  with  $\xi = 0.1, \varepsilon = 0.01$  and  $t = 0.1$ .

$P$	$Nu$
0.7	0.7077
1	1.0100
5	5.0362
10	10.0658

Table-5.4.1 shows the effect of  $Pr$  on Nusselt number  $Nu$  with  $\xi = 0.1, \varepsilon = 0.01$  and  $t = 0.1$ . It is found that, the  $Nu$  increases with increasing  $Pr$ .

**Table –1.4. 2:** Effect of  $\varepsilon$  on Nusselt number  $Nu$  with  $\xi = 0.1, Pr = 1$  and  $t = 0.1$ .

$\varepsilon$	$Nu$
0	1
0.01	1.0100
0.1	1.0999

Table-1.4.2 depicts the effect of  $\varepsilon$  on Nusselt number  $Nu$  with  $\xi = 0.1, Pr = 1$  and  $t = 0.1$ . It is observed that,  $Nu$  increases with increasing  $\varepsilon$ .

**Table – 1.4.3:** Effect of  $t$  on Nusselt number  $Nu$  with  $\xi = 0.1, Pr = 1$  and  $t = 0.01$ .

$T$	$Nu$
0	1.0100
1	1.0098
2	1.0096
3	1.0092
4	1.0088
5	1.0083
6	1.0076
7	1.0069
8	1.0062
9	1.0053
10	1.0044

Table-1.4.3 illustrates the effect of ton Nusselt number  $Nu$  with  $\xi = 0.1, Pr = 1$  and  $\varepsilon = 0.01$ . It is noted that, the  $Nu$  decreases with increasing  $t$ .

## REFERENCES

1. T. Ariman, M.A. Turk and N. D. Sylvester, Microcontinuum fluid mechanics - a review, Int. J. Eng. Sci. 11 (8) (1973), 905-930.
2. T. Ariman, M.A. Truck and N. D. Sylvester, Applications of Microcomtinuum fluid mechanics, Int. J.Eng. Sci. 12 (4) (1974), 273-293.
3. O. B. Beg and S. K. Gosh, Analytical study of MHD Radiation Convection with surface temperature oscillation and secondary flow effects, international journal of Applied Mathematics and Machinery, vol. 6(6) (2009), 1-22.
4. C. Eringen, Theory of micropolar fluids, J. Math. Mech. 16(1) (1966), 1-16.
5. R. S. R. Gorla and R. Tornabene, Free convection from a vertical plate with conuniform surface heat flux and embedded in a porous medium, Transc. Porous Media, 3 (1988), 95-106.
6. Hassanien and F. M. Hady, Hydromagnetic free convection and mass transfer flow of non-Newtonian fluid through a porous medium bounded by an infinite vertical limiting surface with constant suction, Astrophys Space Sci., 116 (1985), 141-148.
7. F. S. Ibrahim and F. M. Hady, Mixed convection over a horizontal plate with vectored mass transfer in a transverse magnetic field, Astrophysics and Space Science, Astrophys Space Sci., 114 (1985), 335.
8. Y. J. Kim, Unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with vatiable suction, Int. J. Engng. Sci. 38 (2000), 833-845.
9. Y. J. Kim, Unsteady convection flow of micropolar fluids past a vertical porous medium, Acta Mech. 148 (1-4) (2001), 105-116.
10. Y. J. Kim and J. C. Lee, Analytical studies on MHD oscillatory flow of a micropolar fluid over a vertical porous plate, Surface and Coatings Technology, 171(2003), 187-193.
11. O. D. Makinde and, P. Y. Mhone, Effects of Heat transfer to MHD oscillatory flow in a channel filled with porous medium, Rom. J. Phys., 50(2005), 931-938.
12. Mostafa, Thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity, The Canadian journal of Chemical Engineering, 87(2009), 171- 181.
13. A.Reddappa, M. V. Subba Reddy and K. R. Krishna Prasad, Thermal effects in Stokes' second problem for unsteady magneto hydrodynamic flow of a Micropolar fluid, Journal of Pure and Applied Physics, Vol. 21, No.3(2009), 365-373.
14. Raptis and N. Kafousias, Heat transfer in flow through a porous medium bounded by an infinite vertical plate under the action of a magnetic field, International Journal of Energy Research, 6(1982), 241-245.
15. Raptis, flow of a micropolar fluid past a continuously moving plate by the presence of radiation, Int. J. Heat Mass Transfer, 41 (1998), 2865-2866.
16. W.R. Schowalter, The application of boundary-layer theory to power-law pseudoplastic fluids: Similarity solutions, AIChE J., 6(1960), 24-28.
17. P.G. Siddheshwar and U. S. Mahabaleshwar, Effects of Radiation and Heat sources on MHD flow of a visco-elastic liquid and Heat transfer over a stretching sheet. International Journal of Non-linear Mechanics, 40(2005), 807-820.
18. A.K. Singh, MHD free convection and mass transfer flow with Hall current, viscous dissipation, joule heating and thermal diffusion, Indian Journal of Pure Applied Physics, 66(2003), 71.
19. V. M. Soundalgekar, Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction, Proc. Roy. Soc. London A, 333 (1973), 25-36.

**Source of Support: Nil, Conflict of interest: None Declared**

**[Copy right © 2016, RJPA. All Rights Reserved. This is an Open Access article distributed under the terms of the International Research Journal of Pure Algebra (IRJPA), which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.]**

---

**Corresponding Author: K. Kashaiah\***

**Department of Mathematics, Osmania University Hyderabad, India.**