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TRAVERSABILITY OF BLOCK LINE GRAPHS

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ABSTRACT

The black line graph $B_l(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G, with two points adjacent if one corresponds to a point of G and other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B. In this paper, we establish a necessary and sufficient condition for the block line graph of a connected graph to be eulerian. Also we obtain a characterization of graphs whose block line graphs are hamiltonian.

Keywords: *block line graph, eulerian graph, hamiltonian graph.*

Mathematics Subject Classification: *05C.*

1. INTRODUCTION

The graphs considered in this paper are finite, undirected without loops and multiple lines. Any undefined term here may be found in Kulli [1].

If $B = \{u_1, u_2,\ldots,u_r, r\geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 of G are incident with a common cut point, then they are adjacent blocks. This idea was introduced by Kulli in [2]. The points, lines and blocks are called its members.

The block line graph $B_l(G)$ of a graph G is the graph whose point set is the union of the set of points, lines and blocks of G, with two points adjacent if one corresponds to a point and other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B. This concept was introduced by Kulli in [3]. Many other graph valued functions in graph theory were studied, for example, in [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18].

In this paper, we establish a characterization of graphs whose block line graphs are eulerian. Also some properties of hamiltonian block line graphs are obtained. Traversability of some graph valued functions were studied, for example, in [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

We consider graphs without isolated points.

The following result will be useful to prove our results.

Theorem A [1, *p*76]: A connected graph G is eulerian if and only if every point of G is the even degree.

2. EULERIAN BLOCK LINE GRAPHS.

Remark 1: If v is point of a graph G and v_1 is the corresponding point of v in $B_l(G)$, then $\deg_{B_l(G)} v_1 = \deg_G v + m$, whose m is the number of blocks containing v.

Remark 2: If e is a line of a graph G and e_1 is the corresponding point of e in $B_l(G)$, then $\deg_{B_l(G)} e_1 = 2$.

Remark 3: If B is a block of a graph G and B_1 is the corresponding point of B in $B_l(G)$, then $\deg_{B_l(G)} B_1 = n$ where n is the number of points in B.

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Theorem 4: Let G be a nontrivial graph. If G is eulerian then $B_l(G)$ is not eulerian.

Proof: Suppose G is a nontrivial eulerian graph. Then G is connected and by Theorem A, every point of G is of even degree. Since every nontrivial connected graph has at least two noncutpoints, it implies that G has a noncutpoint v of even degree. By Remark 1, we see that $\deg_{B_l(G)} v_1$ is odd, where v_1 is the corresponding point of v in $B_l(G)$. Thus by Theorem A, $B_l(G)$ is not eulerian.

A necessary and sufficient condition for a graph whose block line graph is eulerian is presented in the following theorem.

Theorem 5: Let G be a nontrivial connected graph. The block line graph $B_l(G)$ is eulerian if and only if G satisfies the following conditions.

1. each noncutpoint of G is incident with odd number of lines,
2. each cutpoint of G is incident with either even number of lines and even number of blocks or odd number of lines and odd number of blocks, and
3. each block of G is incident with even number of points.

Proof: Suppose $B_l(G)$ is eulerian. Let v be a point of $B_l(G)$. Then v is a point or a line or a block of G. We have the following 3 cases.

Case-1: Suppose v is a point of G. Then by Remark 1,

$$\deg_{B_l(G)} v = \deg_G v + m$$

where m is the number of blocks containing v.

We consider the following two subcases.

Subcase-1: Suppose v is a noncutpoint of G. Then m = 1. By Theorem A, $\deg_{B_l(G)} v$ is even. Hence $\deg_G v$ is odd.

Thus (1) holds.

Subcase-2: Suppose v is a cutpoint of G. By Theorem A $\deg_{B_l(G)} v$ is even. Hence both $\deg_G v$ and m are either even or odd. Thus (2) holds.

Case-2: Suppose v is a line e of G. Then by Remark 2, $\deg_{B_l(G)} v = 2$.

Case-3: Suppose v is a block B of G. Then by Remark 3, deg $\deg_{B_l(G)} B = n$, where n is the number of points in B. By Theorem A, $\deg_{B_l(G)} B$ is even. Thus n is even. Thus (3) holds.

Conversely suppose (1), (2) and (3) hold. Suppose v is a point of $B_l(G)$. Then v is a point or a line or a block of G. If v is a point of G, then v is either a noncutpoint or a cutpoint of G. If v is a noncutpoint of G, then by Condition (1) and Remark 1, $\deg_{B_l(G)} v$ is even. If v is a cutpoint of G, then by Condition (2) and Remark 1, $\deg_{B_l(G)} v$ is even. If v is a line e of G, then by Remark 2, $\deg_{B_l(G)} e$ is even. If v is a block B of G, then by Condition (3) and Remark 3, $\deg_{B_l(G)} B$ is even. Thus every point of $B_l(G)$ is of even degree. By Theorem A, $B_l(G)$ is eulerian.

Corollary 6: If G is a nontrivial path, then $B_l(G)$ is eulerian.

Proof: This follows from Theorem 5.

Corollary 7: If G is a cycle, then $B_l(G)$ is not eulerian.

Proof: This follows from Theorem 4.

3. HAMILTONIAN BLOCK LINE GRAPHS

Remark 8[3]: If v is a cut point in G, then the corresponding point v_1 of v in $B_l(G)$ is also a cutpoint.

Proposition 9: If a connected graph G has a cut point, then $B_l(G)$ is not hamiltonian.

Proof: This follows from Remark 8.

We obtain a characterization of graphs whose block line graphs are hamiltonian.

Theorem 10: The block line graph $B_l(G)$ of G is hamiltonian if and only if G is P_2.

Proof: Suppose G is P_2. Then $B_l(G) = C_4$ and hence $B_l(G)$ is hamiltonian.

Conversely suppose $B_l(G)$ is hamiltonian. We now prove that $G = P_2$. On the contrary, assume $G \neq P_2$. We now consider the following cases.

Case-1: Suppose G is disconnected. Then $B_l(G)$ is disconnected Hence $B_l(G)$ is not hamiltonian.

Case-2: Suppose G is a connected graph with a cutpoint. By Proposition 9, $B_l(G)$ has a cutpoint and hence $B_l(G)$ is not hamiltonian.

Case-3: Suppose G is a block B with $p \geq 3$ points. Then G has a cycle $C_n = v_1\, v_2\, v_3 \ldots\, v_n\, v_1$, $n \geq 3$. In $B_l(G)$,

$C_{2n} = v_1\, e_1\, v_2\, e_2\, \ldots\, e_{n-1}\, v_n\, e_n\, v_1$ is a cycle. Let u be a point in $B_l(G)$ corresponding to the block B. Then u is adjacent with all the points v_i, $1 \leq i \leq n$, in $B_l(G)$ Since every pair of points v_i and v_j are not adjacent in $B_l(G)$, it implies that $B_l(G)$ has a subgraph homeomorphic to $K_{2,3}$. Thus $B_l(G)$ is not Hamiltonian.

Thus from the above 3 cases, we conclude that $G = P_2$.

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