International Research Journal of Pure Algebra-6(1), 2016, 221-226
Available online through www.rjpa.info ISSN 2248–9037

ON SKOLEM MEAN LABLING FOR FOUR STAR

V. BALAJI[1], D. S. T. RAMESH[2] and S. RAMARAO[3]

[2]Department of Mathematics, Margoschis College, Nazareth - 628617, India.

[1, 3]Department of Mathematics, Sacred Heart College, Tirupattur - 635 601, India.

(Received On: 11-12-15; Revised & Accepted On: 13-01-16)

ABSTRACT

In this paper, we prove the conjecture that the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| < 7$ for $m-6 \le n \le m$ and $1 \le m \ge n$.

Keywords: *Skolem mean graph and star.*

2010 Mathematical Subject Classification Number: *05C78.*

1. INTRODUCTION

All graphs in this chapter are finite, simple and undirected. Terms not defined here are used in the sense of Harary **[8]**. In **[2]**, we proved that the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+\ell$ for $\ell = 1,2,3,\ldots,$ $m = 1,2,3,\ldots,$ $n = \ell+m+4$ and $\ell \le m < n$; the three star $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+\ell$ for $\ell = 1,2,3,\ldots,$ $m = 1,2,3,\ldots,$ $n \ge \ell+m+5$ and $\ell \le m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 4+2\ell$ for $\ell = 2,3,4,\ldots,$ $m = 2,3,4,\ldots,$ $n = 2\ell+m+4$ and $\ell \le m < n$; the four star $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 4+2\ell$ for $\ell = 2,3,4,\ldots,$ $m = 2,3,4,\ldots,$ $n = 2\ell+m+5$ and $\ell \le m < n$; the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is a skolem mean graph if $|m-n| = 7$ for $m = 1,2,3,\ldots,$ $n = m+7$ and $1 \le m < n$. Also, the four star $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ is not a skolem mean graph if $|m-n| > 7$ for $m = 1,2,3,\ldots,$ $n \ge m+8$ and $1 \le m < n$. **In [3]**, the necessary condition for a graph to be skolem mean is that $p \ge q+1$.

2. SKOLEM MEAN LABELING

Definition 2.1: The four star is the disjoint union of $K_{1,a}$, $K_{1,b}$, $K_{1,c}$ and $K_{1,d}$. It is denoted by $K_{1,a} \cup K_{1,b} \cup K_{1,c} \cup K_{1,d}$.

**Corresponding Author: V. Balaji*[4]*
[1, 3]Department of Mathematics, Sacred Heart College, Tirupattur - 635 601, India.

Definition 2.2 [2]: A graph $G=(V, E)$ with p vertices and q edges is said to be a skolem mean graph if there exists a function f from the vertex set of G to $\{1,2,3,\dots,p\}$ such that the induced map f* from the edge set of G to $\{2,3,4,\dots,p\}$ defined by

$$f^*(e=uv)=\begin{cases}\dfrac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even}\\ \dfrac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd, then}\end{cases}$$

the resulting edges get distinct labels from the set $\{2,3,4,\dots,p\}$.

Note 2.3: In a skolem mean graph, $p\geq q+1$.

Theorem 2.4: If $1\leq m\geq n$, the four star $K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ is a skolem mean graph if $|m-n|<7$ for $n=1,2,3,\dots$ and $m-6\leq n\leq m$.

Proof:

Case (a) Consider the graph $G=K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ when $n=m$. Let us consider the case that $|m-n|<7$ for $n=1,2,3,\dots$. we have to prove that G is a skolem mean graph.

We have $V(G)=\{u,u_1\},\{v,v_1\},\{w\}\cup\{w_i:1\leq i\leq m\}\cup\{x_j:1\leq j\leq n\}$ and

$$E(G)=\{uu_1,vv_1\}\cup\{ww_i:1\leq i\leq m\}\cup\{xx_j:1\leq j\leq n\}.$$

Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f:V(G)\to\{1,2,3,4,\dots,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=5;\ f(w)=m+n+5;\ f(x)=3;$$
$$f(u_1)=9;$$
$$f(v_1)=7;$$
$$f(w_i)=2i \quad \text{for } 1\leq i\leq m;$$
$$f(x_j)=2j+9 \quad \text{for } 1\leq j\leq n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 5 ; vv_1 is 6 ; ww_i is $\dfrac{m+n+2i+5}{2}$ for $1\leq i\leq m$ and xx_j is $j+6$ for $1\leq j\leq n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(b): Consider the graph $G=K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ when $n=m-1$. Let us consider the case that $|m-n|<7$ for $n=1,2,3,\dots$. We have to prove that G is a skolem mean graph.

We have $V(G)=\{u,u_1\},\{v,v_1\},\{w\}\cup\{w_i:1\leq i\leq m\}\cup\{x_j:1\leq j\leq n\}$.

$$E(G)=\{uu_1,vv_1\}\cup\{ww_i:1\leq i\leq m\}\cup\{xx_j:1\leq j\leq n\}.$$

Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f: V(G) \to \{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=2;\ f(w)=m+n+5;\ f(x)=4;$$
$$f(u_1)=6;$$
$$f(v_1)=8;$$
$$f(w_i)=2i+1 \quad for\ 1 \le i \le m;$$
$$f(x_j)=2j+8 \quad for\ 1 \le j \le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 4 ; vv_1 is 5 ; ww_i is $\dfrac{m+n+2i+6}{2}$ for $1 \le i \le m$ and xx_j is $j+6$ for $1 \le j \le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(c): Consider the graph $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m-2$. Let us consider the case that $|m-n| < 7$ for $n = 1,2,3,...$. We have to prove that G is a skolem mean graph.

We have $V(G) = \{u,u_1\}, \{v,v_1\}, \{w\} \cup \{w_i : 1 \le i \le m\} \cup \{x_j : 1 \le j \le n\}$.
$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_j : 1 \le j \le n\}$.
Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f: V(G) \to \{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=3;\ f(w)=m+n+5;\ f(x)=5;$$
$$f(u_1)=2;$$
$$f(v_1)=7;$$
$$f(w_i)=2i+2 \quad for\ 1 \le i \le m;$$
$$f(x_j)=2j+7 \quad for\ 1 \le j \le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 2 ; vv_1 is 5 ; ww_i is $\dfrac{m+n+2i+7}{2}$ for $1 \le i \le m$ and xx_j is $j+6$ for $1 \le j \le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(d): Consider the graph $G = K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$ when $n = m-3$. Let us consider the case that $|m-n| < 7$ for $n = 1,2,3,...$. We have to prove that G is a skolem mean graph.

We have $V(G) = \{u,u_1\}, \{v,v_1\}, \{w\} \cup \{w_i : 1 \le i \le m\} \cup \{x_j : 1 \le j \le n\}$.
$E(G) = \{uu_1, vv_1\} \cup \{ww_i : 1 \le i \le m\} \cup \{xx_j : 1 \le j \le n\}$.

Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f:V(G)\to\{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=2;\ f(w)=m+n+5;\ f(x)=6;$$
$$f(u_1)=3;$$
$$f(v_1)=4;$$
$$f(w_i)=2i+3 \quad for\ 1\le i\le m;$$
$$f(x_j)=2j+6 \quad for\ 1\le j\le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 2 ; vv_1 is 3; ww_i is $\dfrac{m+n+2i+8}{2}$ for $1\le i\le m$ and xx_j is $j+6$ for $1\le j\le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(e): Consider the graph $G=K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ when $n=m-4$. Let us consider the case that $|m-n|<7$ for $n=1,2,3,...$. We have to prove that G is a skolem mean graph.

We have $V(G)=\{u,u_1\},\{v,v_1\},\{w\}\cup\{w_i:1\le i\le m\}\cup\{x_j:1\le j\le n\}$.

$E(G)=\{uu_1,vv_1\}\cup\{ww_i:1\le i\le m\}\cup\{xx_j:1\le j\le n\}$.

Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f:V(G)\to\{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=2;\ f(w)=m+n+5;\ f(x)=5;$$
$$f(u_1)=3;$$
$$f(v_1)=4;$$
$$f(w_i)=2i+4 \quad for\ 1\le i\le m;$$
$$f(x_j)=2j+5 \quad for\ 1\le j\le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 2 ; vv_1 is 3 ; ww_i is $\dfrac{m+n+2i+9}{2}$ for $1\le i\le m$ and xx_j is $j+5$ for $1\le j\le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(f): Consider the graph $G=K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ when $n=m-5$. Let us consider the case that $|m-n|<7$ for $n=1,2,3,...$. We have to prove that G is a skolem mean graph.

We have $V(G)=\{u,u_1\},\{v,v_1\},\{w\}\cup\{w_i:1\le i\le m\}\cup\{x_j:1\le j\le n\}$.

$E(G)=\{uu_1,vv_1\}\cup\{ww_i:1\le i\le m\}\cup\{xx_j:1\le j\le n\}$.

Then G has $m+n+6$ vertices and $m+n+2$ edges. The required vertex labeling $f:V(G)\to\{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=2;\ f(w)=m+n+5;\ f(x)=5;$$
$$f(u_1)=3;$$
$$f(v_1)=4;$$
$$f(w_i)=2i+5 \quad for\ 1\le i\le m;$$
$$f(x_j)=2j+4 \quad for\ 1\le j\le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 2; vv_1 is 3 ; ww_i is $\frac{m+n+2i+10}{2}$ for $1\le i\le m$ and xx_j is $j+5$ for $1\le j\le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

Case-(g): Consider the graph $G=K_{1,1}\cup K_{1,1}\cup K_{1,m}\cup K_{1,n}$ when $n=m-6$. Let us consider the case that $|m-n|<7$ for $n=1,2,3,\ldots$. We have to prove that G is a skolem mean graph.

We have $V(G)=\{u,u_1\},\{v,v_1\},\{w\}\cup\{w_i:1\le i\le m\}\cup\{x_j:1\le j\le n\}$.

$E(G)=\{uu_1,vv_1\}\cup\{ww_i:1\le i\le m\}\cup\{xx_j:1\le j\le n\}$.

Then G has $m+n+6$ vertices and $m+n+2$ edges.

The required vertex labeling $f:V(G)\to\{1,2,3,4,...,m+n+6\}$ is defined as follows:

$$f(u)=1;\ f(v)=2;\ f(w)=m+n+5;\ f(x)=6;$$
$$f(u_1)=3;$$
$$f(v_1)=4;$$
$$f(w_i)=2i+6 \quad for\ 1\le i\le m;$$
$$f(x_j)=2j+3 \quad for\ 1\le j\le n$$

The corresponding edge labels are as follows:

The edge label of uu_1 is 2 ; vv_1 is 3 ; ww_i is $\frac{m+n+2i+11}{2}$ for $1\le i\le m$ and xx_j is $j+5$ for $1\le j\le n$.

Hence the induced edge labels of G are distinct.

Hence the graph G is skolem mean graph.

3. ACKNOWLEDGEMENT

One of the authors (Dr. V. Balaji) acknowledges University Grants Commission, SERO and Hyderabad, India for financial assistance (No. F MRP 5766 / 15 (SERO / UGC)).

4. REFERENCES

1. J. C. Bermond, Graceful graphs, radio antennae and French windmills, Graph theory and Combinatorics, Pitman, London, (1979), 13 – 37.
2. V. Balaji, D. S. T. Ramesh and A. Subramanian, Skolem Mean Labeling, Bulletin of Pure and Applied Sciences, vol. 26E No. 2, 2007, 245 – 248.
3. V. Balaji, D. S. T. Ramesh and A. Subramanian, Some Results on Skolem Mean Graphs, Bulletin of Pure and Applied Sciences, vol. 27E No. 1, 2008, 67 – 74.
4. V. Balaji, Solution of a Conjecture on Skolem Mean Graph of stars $K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, International Journal of Mathematical Combinatorics, vol.4, 2011, 115 – 117.
5. V. Balaji, D. S. T. Ramesh and V. Maheswari, Solution of a Conjecture on Skolem Mean Graph of Stars $K_{1,\ell} \cup K_{1,\ell} \cup K_{1,m} \cup K_{1,n}$, International Journal of Scientific & Engineering Research, 3(11) 2012,125 - 128.
6. V. Balaji, D. S. T. Ramesh and V. Maheswari, Solution of a Conjecture on Skolem Mean Graph of Stars $K_{1,1} \cup K_{1,1} \cup K_{1,m} \cup K_{1,n}$, Sacred Heart Journal of Science & Humanities, Volume 3, July 2013.
7. J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 6 (2010), # DS6.
8. F. Harary, Graph Theory, Addison – Wesley, Reading, 1969.
9. V. Maheswari, D. S. T. Ramesh and V. Balaji, On Skolem Mean Labeling, Bulletin of Kerala Mathematics Association, Vol. 10, No.1, 2013, 89 – 94.
10. S. Somasundaram and R. Ponraj, Mean labeling of graphs, National Academy Science letters, 26 (2003), 210-213.
11. S. Somasundaram and R. Ponraj, Non – Existence of mean labeling for a wheel, Bulletin of Pure and Applied Sciences (Section E : Mathematics & Statistics), 22E (2003), 103 – 111.
12. S. Somasundaram and R. Ponraj, Some results on mean graphs, Pure & Applied Mathematika Sciences 58 (2003), 29 - 35.

Source of Support: University Grants Commission, SERO and Hyderabad, India.
Conflict of interest: None Declared