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INFINTESIMAL C-CONFORMAL MOTIONS OF FINSLRIRAN HYPERSURFACES

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ABSTRACT

*Let F^{n-1} be an hypersurface of a Finsler space F^n. In this paper we obtain condition for an infinitesimal C-Conformal motion for hypersurface of Finsler space and prove some results relating to the infinitesimal C-Conformal motion. Further a condintion is obtained to preserve *P-Finsler hypersurface and an infintesimal C-Conformal motion preserves C^h-recurrent Finsler hypersurface.*

Key Words: *Finsler Space, *P -Finsler space, P-reducible, C-reducible, C^h-recurrent Finsler spaces.*

AMS Subject Classification (2000): *53B40, 53C60*

1. INTRODUCTION

The theory of conformal changes of Finsler metrics has been initiated by M. S. Knebelman in 1929. H. Izumi ([1], [2]) gave the condition for a Finsler space to be h-conformally flat and he has also studied *P-Finsler space. Later S. Kikuchi [3] gave the condition for a Finsler space to be conformally flat. M. Hashiguchi introduced a special change called C-conformal change which satisfies C-conditions. C. Shibata and M. Azuma have studied C-conformal invariant tensors of Finsler metric. The authors S. K. Narasimhamurthy and C. S. Bagewadi have published a paper on infinitesimal C-conformal motions of Special Finsler spaces [6]. In this we study infinitesimal C-conformal motions on some special Finsler hypersurfaces and obtain some results on *P-Finsler hypersurfaces, C-reducible, P-reducible, C^h-recurrent Finsler hypersurfaces.

2. PRELIMINARY

Let M^n be an n-dimensional smooth manifold and $F^n = (M^n, L)$ be an n-dimensional Finsler space equipped with fundamental function L(x, y) on M^n. The metric tensor $g_{ij}(x, y)$, angular metric tensor h_{ij} and Cartan's C-tensor C_{ijk} are given by

$$g_{ij} = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j L^2, \quad g^{ij} = g_{ij}^{-1}$$
$$h_{ij} = g_{ij} - l_i l_j$$
$$C_{ijk} = \frac{1}{2}\dot{\partial}_k\, g_{ij}$$
$$C_{ij}^k = \frac{1}{2}g^{km}(\dot{\partial}_m g_{ij}), \quad where \quad \dot{\partial}_i = \frac{\partial}{\partial x^i}$$

The conformal change of a Finsler metric $L = \bar{L} = e^{\sigma(x)}L$ then σ is called the conformal factor and depends on point x only. By this change, we have another Finsler space $\bar{F}^n = (M^n, \bar{L})$ on the same underlying manifold M^n. M. Hashiguchi introduced the special change called C-conformal which is non-homethetic conformal change satisfying

$$C_{\alpha\beta}^{\gamma}\sigma^{\alpha} = 0, \quad \sigma^{\alpha} = g^{\alpha\beta}\sigma_{\alpha}, \ \sigma_{\alpha} = \partial\sigma/\partial x^{\alpha} \ ,$$

$$C_{\alpha\beta}^{\gamma}\sigma^{\alpha} = 0, \quad \sigma^{\alpha} = C_{\gamma}^{\alpha\beta}\sigma^{\eta}g_{\eta\beta} = C_{\eta\gamma}^{\alpha}\sigma^{\eta} = 0, \text{ also}$$

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$$C_{\alpha\beta\gamma}\sigma^{\beta} = C^{\delta}_{\alpha\gamma}g_{\beta\delta}\sigma^{\beta} = C^{\delta}_{\alpha\gamma}\sigma_{\delta} = 0. \tag{2.1}$$

The Berwald connection and the Cartan connection of F^n are given

$B\Gamma = \left(G^i_{jk}, N^i_{jk}, 0\right),\ \ C\Gamma = (F^i_{jk}, N^i_j, C^i_{jk})$
respectively.

A hypersurface $F^{n-1}$of the underlying smooth manifold F^n may be parametrically represented by the equation $x^i = x^i(u^{\alpha})$, where u^{α} are Gaussian coordinates on F^{n-1} and Greek indices run from 1 to (n-1). Here we shall assume that the matrix consisting of the projection factors $B^i_{\alpha} = \partial x^i/\partial u^{\alpha}$ is of rank (n-1). The following notations are also employed

$$B^i_{\alpha\beta} = \frac{\partial x^1}{\partial u^{\alpha}\partial u^{\beta}}\ , B^i_{0\beta} = v^{\alpha}B^i_{\alpha\beta},\ \ B^{ij\dots}_{\alpha\beta\dots} = B^i_{\alpha},\ \ B^j_{\beta\dots.}$$

If the supporting element y^i at a point (u^{α}) of M^{n-1} is assumed to be tangential to M^{n-1}, we may then write $y^i = B^i_{\alpha}(u)v^{\alpha}$ i.e F^n is thought of as the supporting element of M^{n-1} at a point (u_{α}). Since the function $\underline{L}(u,v) = L(x(u), y(u,v))$ gives rise to a Finsler matrix of M^{n-1}, we get a (n-1) dimension Finsler space $F^{n-1} = (M^{n-1}, \underline{L}(u,v))$.

At each point (u_{α}) of F^{n-1}, the unit normal vector $N^i(u,v)$ is defined by

$$g_{ij}B^i_{\alpha}N^i = 0,\ \ g_{ij}N^iN^j = 1.$$

If (B^i_{α}, N_i), is the inverse matrix of (B^i_{α}, N^i), we have

$B^i_{\alpha}B^{\beta}_i = \delta^{\beta}_{\alpha},\ B^i_{\alpha}N_i = 0,\ N^iN_i = 1$ and further $B^{\alpha}_iB^{\alpha}_j + N^iN_j = \delta^i_j$.

Making use of the inverse $g^{\alpha\beta}\ of\ \ (g_{\alpha\beta})$), we get

$$B^{\alpha}_i = g^{\alpha\beta}g_{ij}B^j_{\alpha},\ \ N_i = g_{ij}N^j.$$

3. LEI DERIVATIVE ON FINSLER HYPERSURFACE

Consider an infinitesimal extended point transformation in a Finsler space generated by the vector $X = v^i(x)\partial_i$, i.e.

$$\bar{X}^i = x^i + v^i dt,\ \ \bar{y}^i = y^i + \left(\partial_j v^i\right)y^j dt. \tag{3.1}$$

The condition for the above transformation to be infinitesimal conformal motion is that there exists a function σ of x such that

$$L_X g_{\alpha\beta} = 2\sigma(X)g_{\alpha\beta},\ \ L_X g^{\alpha\beta} = -2\sigma(X)g^{\alpha\beta}. \tag{3.2}$$

It is well known that on i.c.m (3.1) satisfies

$$L_X C^{\alpha}_{\beta\gamma} = 0,\quad and\ \ L_X y^i = B^i_{\alpha}(u)v^{\alpha} = 0. \tag{3.3}$$

From this we can easily seen that

$(a)\ \ L_X C_j B^j_{\beta} = L_X C_{\beta} = 0,\quad C_{\alpha} = C^{\alpha}_{\beta\alpha},\qquad L_X C^{\beta} = -2\sigma C^{\alpha}$
$(b)\ \ L_X L = \sigma L$
$(c)\ \ L_X l^{\alpha} = -\sigma l^{\alpha},\ \ L_X l_{\alpha} = \sigma l_{\alpha}$
$(d)\ \ L_X h^{\alpha}_{\beta} = 0,\ \ L_X\left(g^{\alpha\delta}g_{\beta\gamma}\right) = 0$

The commutative formulae involving Lie and Covariant derivatives are given by

$(i)\ \ L_X T^{\alpha}_{\beta||\gamma} - (L_X T^{\alpha}_{\beta})_{||\gamma} = T^{\eta}_{\beta}A^{\alpha}_{\eta\gamma} - T^{\alpha}_{\eta}A^{\eta}_{\beta\gamma} - A^{\eta}_{\gamma}\partial_{\eta}T^{\alpha}_{\beta}$ (3.4)
$(ii)\ L_X H^{\alpha}_{\delta\beta\gamma} = A^{\alpha}_{\delta\beta||\gamma} + A^{\eta}_{\beta}G^{\alpha}_{\delta\gamma\eta} - A^{\alpha}_{\delta\gamma\beta} - A^{\alpha}_{\delta\gamma||\beta} - A^{\eta}_{\beta}G^{\alpha}_{\delta\gamma\eta}$,

Where $A^{\eta}_{\beta} = L_X G^{\eta}_{\beta}$, $\quad A^{\alpha}_{\beta\gamma} = L_X G^{\alpha}_{\beta\gamma} = v^{\alpha}_{||\beta||\gamma} + H^{\alpha}_{\beta\gamma\delta} v^{\delta} + G^{\alpha}_{\beta\gamma\eta} v^{\eta}_{||0}$,

where $\frac{\beta}{\gamma}$ means the interchange of indices β and γ in the foregoing terms and symbol '||' denotes the covariant derivative in Finsler hypersurface.

Since the Lie derivative is commutative with ∂_{γ}, $\dot{\partial}_{\gamma}$ and from (3.1)

$$L_X \gamma^{\alpha}_{\beta\gamma} = \sigma_{\beta}\delta^{\alpha}_{\gamma} + \sigma_{\gamma}\delta^{\alpha}_{\beta} - \sigma^{\alpha} g_{\beta\gamma}, \quad \sigma_{\beta} = \partial_{\beta}\sigma. \tag{3.5}$$

4. AN INFINITESIMAL C-CONFROMAL MOTION ON F^{n-1}

If we impose the C-condition on the vector σ_{β}, i.e,

$$C^{\delta}_{\alpha\beta}\sigma_{\delta} = 0, \tag{4.1}$$

then the transformation is called an infinitesimal C-conformal motion (denoted by an i. C-c. m). We have

$$\begin{aligned} &(i)\ \ L\dot{\partial}_k h^i_j = -h_{jk} l^i - h^i_k l_j \\ &(ii)\ \ L^2\big(\dot{\partial}_k C^{ih}_j \alpha_h\big) = L\dot{\partial}_k\big(LC^{ih}_j \alpha_h\big) - LC^{ih}_j \alpha_h \dot{\partial}_h L = 0, \quad or \quad \big(\dot{\partial}_k C^{ih}_j\big)\alpha_h = 0. \end{aligned} \tag{4.2}$$

Transvecting (4.2) (i) by $B^{\alpha jk}_{i\beta\gamma}$ we get

$$L\dot{\partial}_k h^i_j B^{\alpha}_i B^{jk}_{\beta\gamma} = \big(-h_{jk} l^i - h^i_k l_j\big) B^{\alpha}_i B^{jk}_{\beta\gamma}$$

$$L\dot{\partial}_{\gamma} h^{\alpha}_{\beta} = -h_{\beta\gamma} l^{\alpha} - h^{\alpha}_{\gamma} l_{\beta}.$$

Transvecting (4.2) (ii) by $B^{\alpha\beta}_{ij} B^{kh}_{\gamma\delta}$

$$L^2\big(\dot{\partial}_k C^{ih}_j \alpha_h\big) B^{\alpha\beta}_{ij} B^{kh}_{\gamma\delta} = \big(L\dot{\partial}_k\big(LC^{ih}_j \alpha_h\big) - LC^{ih}_j \alpha_h \dot{\partial}_h L\big) B^{\alpha\beta}_{ij} B^{kh}_{\gamma\delta} = 0$$

$$L^2\big(\dot{\partial}_{\gamma} C^{\alpha\delta}_{\beta} \sigma_{\delta}\big) = L\dot{\partial}_{\gamma}\big(LC^{\alpha\delta}_{\beta}\sigma_{\delta}\big) - LC^{\alpha\delta}_{\beta}\sigma_{\delta}\dot{\partial}_{\delta} L = 0.$$

Using the above calculation, we obtain

$$\begin{aligned} &(a)\ A^{\alpha}_{\beta\gamma} = L_X G^{\alpha}_{\beta\gamma} = \sigma_{\gamma}\delta^{\alpha}_{\beta} + \sigma_{\beta}\delta^{\alpha}_{\gamma} - \sigma^{\alpha} g_{\beta\gamma} \\ &(b)\ \ A^{\alpha}_{\beta} = A^{\alpha}_{0\beta} = L_X G^{\alpha}_{\beta} = \sigma_{\beta} y^{\alpha} + \sigma_0 \delta^{\alpha}_{\beta} - \sigma^{\alpha} y_{\beta}. \end{aligned} \tag{4.3}$$

Hence we have

Theorem 4.1: *An infinitesimal C-conformal motion on F^{n-1} satisfies* $L_X G^{\alpha}_{\beta\gamma} = \sigma_{\gamma}\delta^{\alpha}_{\beta} + \sigma_{\beta}\delta^{\alpha}_{\gamma} - \sigma^{\alpha} g_{\beta\gamma}$

$$L_X G^{\alpha}_{\beta} = \sigma_{\beta} y^{\alpha} + \sigma_0 \delta^{\alpha}_{\beta} - \sigma^{\alpha} y_{\beta}.$$

From (3.4) (i), we have

$$L_X C^{\alpha}_{\beta\gamma||\Gamma} - (L_X C^{\alpha}_{\beta\gamma})_{||\Gamma} = A^{\alpha}_{\eta\Gamma} C^{\eta}_{\beta\gamma} - A^{\eta}_{\beta\alpha} C^{\alpha}_{\eta\gamma} - A^{\eta}_{\gamma\beta} C^{\alpha}_{\beta\eta} - A^{\eta}_{\Gamma}\dot{\partial}_{\eta} C^{\alpha}_{\beta\gamma} \tag{4.4}$$

From (3.2), (3.3) and (4.4), we have

$$\begin{aligned} L_X C^{\alpha}_{\beta\gamma||\Gamma} = {} & \big(\sigma_{\eta}\delta^{\alpha}_{\Gamma} + \sigma_{\Gamma}\delta^{\alpha}_{\eta} - \sigma^{\alpha} g_{\eta\Gamma}\big) C^{\eta}_{\beta\gamma} - \Big(\sigma_{\beta}\delta^{\eta}_{\Gamma} + \sigma_{\Gamma}\delta^{\eta}_{\beta} - \sigma^{\eta} g_{\beta\Gamma}\Big) C^{\alpha}_{\eta\gamma} - \big(\sigma_{\gamma}\delta^{\alpha}_{\Gamma} + \sigma_{\gamma}\delta^{\eta}_{\gamma} - \sigma^{\eta} g_{\gamma\Gamma}\big) C^{\alpha}_{\beta\eta} - \\ & \big(\sigma_{\Gamma} y^{\eta} + \sigma_0 \delta^{\eta}_{\Gamma} - \sigma^{\eta} y_{\Gamma}\big)\dot{\partial}_{\eta} C^{\alpha}_{\beta\gamma}, \end{aligned}$$

$$\begin{aligned} L_X C^{\alpha}_{\beta\gamma||\Gamma} &= \sigma_{\Gamma} C^{\alpha}_{\beta\gamma} - \sigma^{\alpha} C_{\Gamma\beta\gamma} - \sigma_{\beta} C^{\alpha}_{\Gamma\gamma} - \sigma_{\Gamma} C^{\alpha}_{\beta\gamma} - \sigma_{\gamma} C^{\alpha}_{\beta\Gamma} - \sigma_{\Gamma} C^{\alpha}_{\beta\gamma} - \sigma_0 \dot{\partial}_{\Gamma} C^{\alpha}_{\beta\gamma} - \sigma_{\Gamma} y^{\eta}\big(\dot{\partial}_{\eta} C^{\alpha}_{\beta\gamma}\big) + \sigma^{\eta} y_{\Gamma}\big(\dot{\partial}_{\eta} C^{\alpha}_{\beta\gamma}\big), \\ L_X C^{\alpha}_{\beta\gamma||\Gamma} &= -\sigma^{\alpha} C_{\Gamma\beta\gamma} - \sigma_{\beta} C^{\alpha}_{\Gamma\gamma} - \sigma_{\gamma} C^{\alpha}_{\beta\gamma} - \sigma_0\big(\dot{\partial}_{\Gamma} C^{\alpha}_{\beta\gamma}\big). \end{aligned} \tag{4.5}$$

Transvecting above by y^{Γ}, we have

$L_X C^{\alpha}_{\beta\gamma||\Gamma} y^{\Gamma} = -\sigma C_{\Gamma\beta\gamma} y^{\Gamma} - \sigma_{\beta} C^{\alpha}_{\Gamma\gamma} y^{\Gamma} - \sigma_{\gamma} C^{\alpha}_{\beta\gamma} y^{\Gamma} - \sigma_0 \big(\dot{\partial}_{\Gamma} C^{\alpha}_{\beta\gamma}\big) y^{\Gamma},$

$$L_X C^{\alpha}_{\beta\gamma||0} = \sigma_0 C^{\alpha}_{\beta\gamma}. \tag{4.6}$$

Thus we have

Theorem 4.2: *An infinitesimal C-conformal motion satisfies* $L_X C^{\alpha}_{\beta\gamma||0} = \sigma_0 C^{\alpha}_{\beta\gamma}$

Definition [4.1]: If the tensor $*\, P^i_{jk} = p^i_{jk} - \alpha_0(x) C^i_{jk}$

Transvecting above equation by $B^{\alpha}_i B^{jk}_{\beta\gamma}$ we get

$*\, P^i_{jk} B^{\alpha}_i B^{jk}_{\beta\gamma} = p^i_{jk} B^{\alpha}_i B^{jk}_{\beta\gamma} - \alpha_0(x) C^i_{jk} B^{\alpha}_i B^{jk}_{\beta\gamma},$

$$*\, P^{\alpha}_{\beta\gamma} = P^{\alpha}_{\beta\gamma} - \sigma_0(x) C^{\alpha}_{\beta\gamma}. \tag{4.7}$$

vanishes, then the space is called a $*\, P$-Finsler hypersurface, $\sigma_0(x)$ is a scalar function. Taking Lie derivative on (4.7)

$L_X *\, P^{\alpha}_{\beta\gamma} = L_X P^{\alpha}_{\beta\gamma} - L_X \sigma_0(x) C^{\alpha}_{\beta\gamma}$

By $*\, P$-condition is given by $P^{\alpha}_{\beta\gamma} = \sigma_0 C^{\alpha}_{\beta\gamma}$ and using (4.6) we get

$L_X *\, P^{\alpha}_{\beta\gamma} = \sigma_0 C^{\alpha}_{\beta\gamma} - L_X(\sigma_0) C^{\alpha}_{\beta\gamma} - \sigma_0 L_X C^{\alpha}_{\beta\gamma},$

$L_X *\, P^{\alpha}_{\beta\gamma} = \sigma_0 C^{\alpha}_{\beta\gamma} - \sigma_0 C^{\alpha}_{\beta\gamma},$

$L_X *\, P^{\alpha}_{\beta\gamma} = 0, \quad where \;\; L_X(\sigma_0) = \sigma_0.$

Hence we state that

Theorem 4.3: *An infinitesimal C-conformal motion preserves* $*\, P$*-Finsler hypersurface.*

Definition [4.2]: If the tensor $*\, P_{ijk} = P_{ijk} - (P_k h_{ij} + P_i h_{jk} + P_j h_{ki})$

Transvecting on both side of the above equation by $B^{\alpha}_i B^{jk}_{\beta\gamma}$

$*\, P_{ijk} B^{\alpha}_i B^{jk}_{\beta\gamma} = P_{ijk} B^{\alpha}_i B^{jk}_{\beta\gamma} - (P_k h_{ij} + P_i h_{jk} + P_j h_{ki}) B^{\alpha}_i B^{jk}_{\beta\gamma},$

$$*\, P_{\alpha\beta\gamma} = P_{\alpha\beta\gamma} - (P_{\gamma} h_{\alpha\beta} + P_{\alpha} h_{\beta\gamma} + P_{\beta} h_{\gamma\alpha}). \tag{4.8}$$

Vanishes, then the Finsler space F^n is called a P-reducible Finlser hypersurface.

Taking Lie derivative on both side of the above equation, we have

$L_X \; *\, P_{\alpha\beta\gamma} = L_X P_{\alpha\beta\gamma} - L_X (P_{\gamma} h_{\alpha\beta} + P_{\alpha} h_{\beta\gamma} + P_{\beta} h_{\gamma\alpha}),$

using $L_X P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma}, \qquad L_X P_{\gamma} = -\sigma P_{\gamma}$,

$L_X \; *\, P_{\alpha\beta\gamma} = L_X P_{\alpha\beta\gamma} - (L_X P_{\gamma} h_{\alpha\beta} + P_{\gamma} L_X h_{\alpha\beta} + L_X P_{\alpha} h_{\beta\gamma} + P_{\alpha} L_X h_{\beta\gamma} + L_X P_{\beta} h_{\gamma\alpha} + P_{\beta} L_X h_{\gamma\alpha}), \quad L_X \; *\, P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [(-\sigma P_{\gamma} h_{\alpha\beta}) + (-\sigma P_{\beta} h_{\gamma\alpha}) + (-\sigma P_{\alpha} h_{\beta\gamma}) + 2\sigma(P_{\alpha} h_{\beta\gamma} + P_{\beta} h_{\gamma\alpha} + P_{\gamma} h_{\alpha\beta})],$

$L_X \; *\, P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [-\sigma(P_{\gamma} h_{\alpha\beta} + P_{\alpha} h_{\beta\gamma} + P_{\beta} h_{\gamma\alpha}) + 2\sigma(P_{\gamma} h_{\alpha\beta} + P_{\alpha} h_{\beta\gamma} + P_{\beta} h_{\gamma\alpha})],$
$L_X \; *\, P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [-\sigma P_{\alpha\beta\gamma} + 2\sigma P_{\alpha\beta\gamma}],$

$L_X \; *\, P_{\alpha\beta\gamma} = 0.$
Thus we state

Theorem 4.4: *An infinitesimal C-conformal motion preserves P-reducible Finsler hypersurface.*

Definition [4.3]: If the tensor

$* C^i_{jkl} = (C^i_{jk:l} - \alpha_l C^i_{jk}),$

Transvecting above equation by $B^\alpha_i B^{jkl}_{\beta\gamma\Gamma}$ we get

$* C^i_{jkl} B^\alpha_i B^{jkl}_{\beta\gamma\Gamma} = (C^i_{jk:l} - \alpha_l C^i_{jk}) B^\alpha_i B^{jkl}_{\beta\gamma\Gamma},$

$$* C^\alpha_{\beta\gamma\Gamma} = (C^\alpha_{\beta\gamma||\Gamma} - \sigma_\Gamma C^\alpha_{\beta\gamma}) \tag{4.9}$$

vanishes, then the space F^n is called C^h-recurrent Finsler hypersurface here $\sigma_\Gamma = \sigma_\Gamma\,(x, y)$ is a covariant vector field. Transvecting (4.5) by σ^Γ and using (2.1) we get

$(L_X C^\alpha_{\beta\gamma||\Gamma})\sigma^\Gamma = -\sigma C_{\Gamma\beta\gamma}\sigma^\Gamma - \sigma_\beta C^\alpha_{\Gamma\gamma}\sigma^\Gamma - \sigma_\gamma C^\alpha_{\beta\Gamma}\sigma^\Gamma - \sigma_0\big(\dot\partial_\Gamma C^\alpha_{\beta\gamma}\big)\sigma^\Gamma,$

$(L_X C^\alpha_{\beta\gamma||\Gamma})\sigma^\Gamma = -\sigma^\alpha C_{\beta\gamma} - \sigma_\beta C^\alpha_\gamma - \sigma_\gamma C^\alpha_\beta - \sigma_0\big(C^\alpha_{\beta\gamma}\big),$

$$(L_X C^\alpha_{\beta\gamma||\Gamma})\sigma^\Gamma = 0. \tag{4.10}$$

Taking Lie-derivative on (4.9), we obtain

$L_X * C^\alpha_{\beta\gamma\Gamma} = (L_X C^\alpha_{\beta\gamma||\Gamma} - L_X \sigma_\Gamma C^\alpha_{\beta\gamma}),$

$L_X * C^\alpha_{\beta\gamma\Gamma} = L_X C^\alpha_{\beta\gamma||\Gamma} - \sigma_\Gamma L_X C^\alpha_{\beta\gamma} - L_X(\sigma_\Gamma) C^\alpha_{\beta\gamma},$

$$L_X * C^\alpha_{\beta\gamma\Gamma} = L_X C^\alpha_{\beta\gamma||\Gamma} - L_X(\sigma_\Gamma) C^\alpha_{\beta\gamma}, \tag{4.11}$$

Transvecting (4.11) by $\sigma^\Gamma$and using (4.10), we have

$L_X * C^\alpha_{\beta\gamma\Gamma}\sigma^\Gamma = L_X C^\alpha_{\beta\gamma||\Gamma}\sigma^\Gamma - L_X(\sigma_\Gamma) C^\alpha_{\beta\gamma}\sigma^\Gamma,$

$L_X * C^\alpha_{\beta\gamma} = L_X C^\alpha_{\beta\gamma||0}\sigma^\Gamma - L_X(\sigma_\Gamma) C^\alpha_{\beta\gamma}\sigma^\Gamma\,,$

$$L_X * C^\alpha_{\beta\gamma} = -\sigma^2 C^\alpha_{\beta\gamma}. \tag{4.12}$$

Thus we obtain in view of Deicke's theorem

Theorem 4.5: *An infinitesimal C-conformal motion preserves C^h-recurrent Finsler hypersurface if and only if it is Riemannian.*

Definition [4.4]: If the tensor

$* C_{ijk} = C_{ijk} - \frac{1}{(n+1)}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j),$

Contracting on both side of the above equation by $B^i_\alpha B^{jk}_{\beta\gamma}$ we get

$* C_{ijk} B^i_\alpha B^{jk}_{\beta\gamma} = C_{ijk} B^i_\alpha B^{jk}_{\beta\gamma} - \frac{1}{n}(h_{ij}C_k + h_{jk}C_i + h_{ki}C_j) B^i_\alpha B^{jk}_{\beta\gamma}\,,$

$$* C_{\alpha\beta\gamma} = C_{\alpha\beta\gamma} - \frac{1}{n}(h_{\alpha\beta}C_\gamma + h_{\beta\gamma}C_\alpha + h_{\gamma\alpha}C_\beta) \tag{4.13}$$

vanishes, then the Finsler space is called a C-reducible Finsler hypersurface.

Taking Lie derivative of (4.13) and using

$L_X C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma}, \qquad L_X h_{ij} = 2\alpha h_{ij}, \quad L_X C_\alpha = 0,$ we get

$L_X * C_{\alpha\beta\gamma} = L_X C_{\alpha\beta\gamma} - \frac{1}{n}(L_X h_{\alpha\beta} C_\gamma + L_X h_{\beta\gamma} C_\alpha + L_X h_{\gamma\alpha} C_\beta),$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n}(L_X h_{\alpha\beta} C_\gamma + L_X h_{\beta\gamma} C_\alpha + L_X h_{\gamma\alpha} C_\beta),$$
$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n}(2\sigma h_{\alpha\beta} C_\gamma + 2\sigma h_{\beta\gamma} C_\alpha + 2\sigma h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n} 2\sigma(h_{\alpha\beta} C_\gamma + h_{\beta\gamma} C_\alpha + h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - 2\sigma C_{\alpha\beta\gamma},$$

$$L_X * C_{\alpha\beta\gamma} = 0.$$

Thus, we have

Theorem 4.6: *An infinitesimal C-conformal motion preserves C-reducible Finsler hyperSurface.*

CONCLUSION:

In this paper, we have applied the Lie derivatives on Finsler hypresurface to obtain the condition for the transformation (3.1) to be infinitesimal Conformal motion and ensures (3.5) as Lie derivative is commutative with $\partial_r, \dot{\partial}_r$. Further we proved that an infinitesimal C-conformal motion preserves *P-Finsler hypersuface, P-reducible Finsler hypersurface and C-reducible Finsler hypersurfae. Also by the view of Deicks's theorem, on the condition of Riemannian we proved that infinitesimal C-conformal motion preserves C^h-recurrent Finsler hypersurface.

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