



**SIMPLE ANNUITIES VERSUS ANNUITIES SCHEME
WITH GEOMETRIC GRADIENT WHAT IS THE BEST INVESTMENT CHOICE?**

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ABSTRACT

In this paper shows a mathematical modeling with two hypothetical scenarios in which there are savings through a scheme simple annuity and another with the same scheme but including a geometric gradient. In both scenarios the term is one year (n), with periodic deposits of \$ 5,000.00 (rp) every 28 days (m) and a nominal interest rate of 1.15% (i) and geometric gradient 5% nominal (Gg) apportioned among 12 deposits. It is expected to know which of the two scenarios represents the best investment.

Keywords: *Geometric gradient, simple annuities, saving and investment.*

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INTRODUCTION

Nowadays the subject of financial education is being discussed in various academic fields as well as being a subject of global agenda. In the last few years it has increased the interest to know what is happening with people who have access to the financial system. Some studies such as Lusardi, Mitchell and Curto (2010), Mandell and Schmid (2009) and the documents themselves G20, UNESCO (2015), Development Bank of Latin America (2013) among others, have generated information to know understand the financial behavior of people.

The use of financial services by the population is related to the demand for loans and savings and investment instruments. In Mexico the amount of financial resources, i.e. the funds of financial savings that involve assets and securities held by individuals and corporations, have had a considerable increase from year 2000, because the total resources increased at an annual average rate of over 11%. It is even more remarkable is that the greatest growth was experienced by domestic savings, with an average annual rate of 13%, against 9.62% in external financing.

This situation becomes even more important because it has allowed increase the proportion of domestic financial resources relative to GDP. In the year 2000, this proportion was around of 36% and, after a period of relative stability, from 2005, year after year, it has increased to reach 61% of GDP in 2011 (Góngora, 2013).

From this growing interest of the Mexican population to have access to services offered by financial institutions for savings and investment, the analysis of this work aims to serve as reference in investment decisions that are looking for increase profitability.

Hence, this study aims to carry out a financial modeling under the hypothetical assumption of an annuity investment fund with constant and similar periodic deposits over time " n " with a reference interest rate " i " and capitalization " m " every 28 days and the corresponding comparative, through a series of deposits which increases in a form of geometric gradient, with the same data, only the new variable including the geometric gradient.

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The aim is to compare models, considering that, in the case of geometric gradient is proportionately a little higher than inflation, taking as reference the immediate previous year, i.e., 2014.

DESIGN AND METHOD

To understand the concept of geometric gradient (Gg) we refer what García-Santillán (2014) points out, who defines it as a series of payments (rents) periodic or flows that increase with constant percentages and consecutive periods of savings, rather than increases constant of money. The cash flows (quotas) change at the same percentage between each period.

The features of this type of annuity with geometric gradient are:

- ✓ Different prepaid deposits are made in each interval savings
- ✓ The start dates and expiration of the annuity or periodic payments are known since the agreement is signed.
- ✓ The capitalizations coincide with the range of savings
- ✓ The time limit begins with signing of the agreement

The variables used to calculate the geometric gradient are:

- ✓ Gg : is the percentage that increase or decrease every deposit
- ✓ Rp_1 : the first deposit.
- ✓ i : nominal interest rate
- ✓ m : Capitalization which can be monthly, bimonthly etc.
- ✓ i / m : is the nominal rate compounded and frequency of payments.
- ✓ n : long-term years (number of periodic deposits)

The first hypothetical scenario presented in this mathematical modeling corresponds to savings scheme with regular payments in a bank account. This scenario suggests deposit of \$5,000.00 monthly; with reference to the interest rate provided by financial institutions in July 2015 (depending on the accumulated amount and periodicity of the term/time). By the method of Early Annuity, one year cumulative amount is performed, as well as geometric gradient method. The result determines the feasibility of using one or another method within the deadline.

The interest rate assigned for the days 9 to 15th of July 2015 with amounts of \$5,000.00 to \$9,999.00 is 1.15% nominal ordinary compounded every 28 days. According to the previous data, the future value of the twelve deposits of \$5,000 shall be determined by the annuity early simple method. Considering the above data, the development is as follows:

Firstly we calculate i as follow:

$$i = \frac{.0115}{360}(28) = .0008944$$

After this, we utilized the formula:

$$Fv = Pr \left(1 + \frac{i}{m} \right)^{\left(\frac{\left(1 + \frac{i}{m} \right)^{n/m} - 1}{i/m} \right)} \quad (1)$$

Where:

Fv : future value; Pr : periodical rent; i : interest rate; n/m : capitalizations

$$Fv = \$5,000.00(1 + .0008944) \left(\frac{(1 + .0008944)^{12} - 1}{.0008944} \right) \quad (1.1)$$

$$Fv = \$5,000.00(1.0008944) \left(\frac{.0107858}{.0008944} \right) \quad (1.2)$$

$$Fv = \$5,004.4720(12.0592576) \quad (1.3)$$

$$Fv = \$60,350.22 \quad (1.4)$$

In order to verify previous calculus, we proceed to use financial software to run an investment fund calculus, and we obtain data shown in Table 1.

Table-1: Saving trough annuity simple

Deposit	Annuity	Interest	Balance
1	\$5,000.00	\$4.48	\$5,004.48
2	\$5,000.00	\$8.95	\$10,013.43
3	\$5,000.00	\$13.44	\$15,026.87
4	\$5,000.00	\$17.93	\$20,044.79
5	\$5,000.00	\$22.42	\$25,067.21
6	\$5,000.00	\$26.91	\$30,094.12
7	\$5,000.00	\$31.41	\$35,125.53
8	\$5,000.00	\$35.91	\$40,161.45
9	\$5,000.00	\$40.42	\$45,201.87
10	\$5,000.00	\$44.93	\$50,246.80
11	\$5,000.00	\$49.45	\$55,296.25
12	\$5,000.00	\$53.97	\$60,350.22
Total deposits			\$60,000.00
Won interests			\$350.22
Final Balance			\$60,350.22

Source: own

To the second approach, the inflation rate in 2014 was taken as a reference to determine the increase the periodical payments that shall be made. Inflation in 2014 was 4.08%, and it is used as basis for calculation; also a percentage of 5% for nominal increases (prorated among the twelve deposits) is determined. This percentage will be higher than inflation due to economic movements and as a protection measure of the fund.

DATA

Agg: Gradient geometric amount

gg = gradient geometric 5% nominal (prorated among twelve deposits)

Pr: periodical rent \$5,000.00

i: interest rate 1.15% nominal

n = 12 (twelve deposits)

n/m: capitalizations every 28 days

For the second scenario, mathematical modeling is as follows:

$$gg = \left(\frac{.05}{360} \right) * 28 = .0038889 \quad \text{and} \quad i = \left(\frac{.0115}{360} \right) * 28 = .0008944$$

$$Agg = Pr(1+i) \left(\frac{(1+i)^{n/m} - (1+gg)^{n/m}}{(i/n) - gg} \right) \quad (2)$$

$$Agg = \$5,000.00(1+.0008944) \left(\frac{(1+.0008944)^{12} - (1+.0038889)^{12}}{.0008944 - .0038889} \right) \quad (2.1)$$

$$Agg = \$5,000.00(1.0008944) \left(\frac{1.0107858 - 1.0476780}{-.0029945} \right) \quad (2.2)$$

$$Agg = \$5,004.4720 \left(\frac{-.0368922}{-.0029945} \right) \quad (2.3)$$

$$Agg = \$5,004.4720(12.3199866)$$

$$Agg = \$61,655.02 \quad (2.4)$$

Table-2: Saving fund through simple annuity with geometric gradient

Deposit	Annuity	Interest	Balance
1	\$5,000.00	\$4.47	\$5,004.47
2	\$5,019.44	\$8.97	\$10,032.88
3	\$5,038.96	\$13.48	\$15,085.33
4	\$5,058.56	\$18.02	\$20,161.90
5	\$5,078.23	\$22.57	\$25,262.71
6	\$5,097.98	\$27.15	\$30,387.85
7	\$5,117.81	\$31.76	\$35,537.41
8	\$5,137.71	\$36.38	\$40,711.50
9	\$5,157.69	\$41.03	\$45,910.21
10	\$5,177.75	\$45.69	\$51,133.65
11	\$5,197.88	\$50.38	\$56,381.92
12	\$5,218.10	\$55.10	\$61,655.11
Total deposits			\$61,300.12
Won interests			\$355.00
Final Balance			\$61,655.12

Source: own

CONCLUSION

When performing calculations using simple annuity and geometric gradient methods, the following results were obtained:

The amount that will be received at the end of the twelve periods using the method of annuity is \$ 60,350.22, of which; \$ 60,000.00 belongs to the total deposits, and the remaining \$350.22, is the total of interest earned during the 12 months.

Using the method of geometric gradient, the amount obtained after 12 deposits including interests earned is \$61,655.12, of which \$61,300.12 were obtained with the total deposits and the remaining \$355.00 are the interest generated during this period.

Therefore, after comparing the results of both methods, we may say that the gradient geometric method is the most suitable, because this method considers the inflation rate, which is necessary to preserve the purchasing power of money over time. Thus, it is convenient to consider this factor when carrying out a saving, as a result, the saved amount will not lose their purchasing power.

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