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NANO SEMI – GENERALIZED CONTINUOUS MAPS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to introduce and study the concepts of new class of maps, namely nanosemi-generalized continuous maps in nano topological spaces. We derive their characterizations in terms of nano semi-generalized closed sets, nano semi-generalized closure and nano semi-generalized interior and obtain some of their interesting properties.

Keywords: *Nano sg-closed sets, Nano sg-open sets, Nano continuity, Nano sg-continuous functions.*

1. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. In 1987, P. Bhattacharyya *et.al.* [2] have introduced the notion of semi-generalized closed sets in topological spaces. In 1990, S.P.Arya *et.al.* [1] have introduced the concept of generalized semi-closed sets to characterize the S-normality axiom. The concept of semi-generalized mappings was studied by R. Devi *et.al.* [5] in 1993.The notion of nano topology was introduced by Lellis Thivagar [7] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He also established and analysed the nano forms of weakly open sets such as nano α -open sets, nano semi-open sets and nano pre-open sets. The aim of this paper is to define and analyse the properties of nano semi-generalized continuity. We also establish various forms of continuities associated to nano semi-generalized closed sets.

2. PREMILINARIES

Definition: 2.1[2] A subset A of of a space (X, τ) is called a semi-generalized closed set if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open.

Definition: 2.2[6] The semi-generalized closure of a subset A of a space X is the intersection of all sg-closed sets containing A and is denoted by sgCl(A).

Definition:2.3[6]The semi-generalized interior of a subset A of a space X is the union of all sg-open sets contained in A and is denoted by sgInt(A).

Definition: 2.4 [**10**] A function f: X→Y is semi -generalized continuous (sg-continuous) if f^{-1}(V) is sg-closed set in X for every closed set V of Y, or equivalently, a function f: X→Y is sg -continuous if and only if the inverse image of each open set is sg-open set.

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Definition: 2.5[7] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

(i) The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.$L_R(X)=U\{R(x):R(x) \subseteq X, x\epsilon U\}$ where R(x) denotes the equivalence class determined by $x\epsilon U$.
(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = U\{R(x): R(x) \cap X\neq\Phi, x\epsilon U\}$.
(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not–X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property: 2.6[7] If (U, R) is an approximation space and X, $Y\subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\Phi) = U_R(\Phi) = \Phi$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X\cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X\cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R [U_R(X)]= U_R(X)$
11. $L_R[L_R(X)] = U_R [L_R(X)]= L_R(X)$.

Definition:2.7 [7] Let U be the universe, R be an equivalence relation on U and the Nano topology $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then by property 2.5, $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. (U, $\tau_R(X)$) is called the Nano topological space. Elements of the Nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets with $[\tau_R(X)]^c$ being called Dual Nano topology of $\tau_R(X)$.If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set B = { U, $L_R(X)$, $B_R(X)$} is the basis for $\tau_R(X)$.

Definition:2.8 [7]If $(U,\tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by NInt(A). NInt(A) is the largest nano open subset of A.
(ii) The nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A.

Remark: 2.9[8] Throughout this paper, U and V are non-empty, finite universes; $X \subseteq U$ and $Y\subseteq V$; U/R and V/R' denote the families of equivalence classes by equivalence relations R and R' respectively on U and V. $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ are the Nano topological spaces with respect to X and Y respectively.

Definition: 2.10[8] A subset A of a Nano topological space $(U,\tau_R(X))$ is said to be nano dense if NCl(A)=U**.**

Definition: 2.11[3] If $(U,\tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

(i) The nano semi-closure of A is defined as the intersection of all nano semi-closed sets containing A and is denoted by NsCl(A). NsCl(A) is the smallest nano semi-closed set containing A and NsCl(A)$\subseteq$A**.**
(ii) The nano semi-interior of A is defined as the union of all nano semi-open subsets of A and is denoted by NsInt(A). NsInt(A) is the largest nano semi open subset of A and NsInt(A)$\subseteq$A**.**

Definition: 2.12[3] A subset A of $(U,\tau_R(X))$ is called nano semi-generalized closed set (Nsg-closed) if NSCl(A)$\subseteq$V and A$\subseteq$V and V is nano semi-open in $(U,\tau_R(X))$.The subset A is called nano sg-open in $(U,\tau_R(X))$ if A^c is nanosg-closed.

Definition: 2.13[8] Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano open set in V is nano open in U.

3. NANO SG-CONTINUITY

In this section, we introduce nano semi-generalized continuous maps (Nsg-continuous maps) in Nano topological spaces. We discuss certain characterizations of Nsg-continuous maps.

Definition:3.1 If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A ⊆ U, then

(i) The nano semi- generalized closure of A is defined as the intersection of all nano semi-generalized closed sets containing A and is denoted by NsgCl(A). NsgCl(A) is the smallest nano semi-generalized closed set containing A and if A is a nano sg-closed set, then NsgCl(A) = A.

(ii) The nano semi-generalized interior of A is defined as the union of all nano semi-generalized open subsets of A and is denoted by NsgInt(A). NsgInt(A) is the largest nano semi-generalized open subset of A. If A is nanosg-open set, then NsgInt(A)= A.

Definition: 3.1 Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous on U if the inverse image of every nano open set in V is nano sg-open in U.

Example: 3.2 Let U ={a, b, c, d} with U/R ={{a},{c},{b, d}}.Let X={a, b}⊆U. Then $\tau_R(X)$ ={U, ϕ,{a},{a, b, d}, {b, d}} which are nano open sets.

Nano sg-open sets are {U, ϕ, {a, c, d}, {a, b, d}, {a, b, c},{b, c, d}, {b, d}, {a, d}, {a, b}, {a, c}, {a}, {b}, {d}}
Nano sg-closed sets are {U, ϕ, {a, c, d}, {a, b, c},{b, c, d}, {b, d}, {c, d},{b, c},{a, c}, {a}, {b}, {c}, {d}}

Let V ={x, y, z, w} with V/R' = {{x},{y, z},{w}}.Let Y={x, z}⊆V. Then $\tau_{R'}(Y)$ ={V, ϕ,{x},{y, z},{x, y, z}} which are nano open sets. Nano sg-open sets are {V, ϕ, {y, z, w}, {x, z, w}, {x, y, w},{x, y, z}, {y, z}, {x, w}, {x, y}, {x, z}, {x}, {y}, {z}} Nano sg-closed sets are {V, ϕ, {x}, {y},{z}, {w}, {x, w},{y, z},{z, w}, {y, w}, {y, z, w}, {x, z, w}, {x, y, w}}
Then define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a) = y, f(b) = x, f(c) = w, f(d) = z. Then f^{-1}(V) = U, $f^{-1}(\phi) = \phi$, f^{-1}({y, z}) = {a, d}, f^{-1}({x}) = {b}, f^{-1}({x, y, z}) = {a, b, d}. Thus the inverse image of every nano open set in V is nano sg-open in U. Hence $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nanosg–continuous.

Theorem: 3.3 A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nanosg-continuous if and only if the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is nano sg-closed in $(U, \tau_R(X))$.

Proof: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano sg-continuous and F be nano closed set in $(V, \tau_{R'}(Y))$. That is, $V - F$ is nano open set in V .Since f is nano sg-continuous, the inverse image of every nano open set in V is nanosg-open in U. Hence $f^{-1}(V - F)$ is nano sg-open in U. That is, $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is nano sg-open in U. Hence $f^{-1}(F)$ is nano sg-closed in U. Thus the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ is nano sg-closed in $(U, \tau_R(X))$ if f is nano sg-continuous.

Conversely, let the inverse image of every nano closed set in $(V, \tau_{R'}(Y))$ be nano sg-closed in $(U, \tau_R(X))$. Let H be a nano open set in V. Then $V - H$ is nano closed in V and $f^{-1}(V - H)$ is nano sg-closed in U. That is, $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(H)$ is nano sg-closed in U. Hence $f^{-1}(H)$ is nano sg-open in U. Thus the inverse image of every nano open set in $(V, \tau_{R'}(Y))$ is nano sg-open in $(U, \tau_R(X))$ .This implies that $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous on U.

Theorem: 3.4 Every nano continuous map is nano sg-continuous but not conversely.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano continuous on U. Also every nano closed set is nano sg-closed but not conversely. Since f is nano continuous on $(U,\tau_R(X))$, the inverse image of every nano closed set in $(V,\tau_{R'}(Y))$ is nano closed in $(U,\tau_R(X))$ .Hence the inverse image of every nano closed set in V is nano sg-closed in U and so $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous.

Conversely, all nano sg-closed sets are not nano closed sets and hence nano sg-continuous map is not nano continuous.

Theorem: 3.5 A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous if and only if $f(NsgCl(A))\subseteq NCl(f(A))$ or every subset A of $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano sg-continuous and A $\subseteq$ U. Then $f(A)\subseteq V$. Hence $NCl(f(A))$ is nano closed in V. Since f is nano sg-continuous, $f^{-1}(NCl(f(A))$ is also nano sg-closed in $(U,\tau_R(X))$ .Since $f(A)\subseteq NCl(f(A))$, we have $A\subseteq f^{-1}(NCl(f(A))$.Thus $f^{-1}(NCl(f(A))$ is a nano sg-closed set containing A. But $NsgCl(A)$ is the smallest nano sg-closed set containing A. Hence we have $NsgCl(A)\subseteq f^{-1}(NCl(f(A))$ which implies $f(NsgCl(A))\subseteq NCl(f(A))$.

Conversely, let $f(NsgCl(A))\subseteq NCl(f(A))$ for every subset A of $(U,\tau_R(X))$. Let F be a nano closed set in $(V,\tau_{R'}(Y))$.Now $f^{-1}(F)\subseteq U$, hence, $f(NsgCl(f^{-1}(F)))\subseteq NCl(f(f^{-1}(F)))=NCl(F)$. That is, $NsgCl(f^{-1}(F))\subseteq f^{-1}(NCl(F))=f^{-1}(F)$ as F is nano closed. Hence $NsgCl(f^{-1}(F))\subseteq$ $f^{-1}(F)\subseteq NsgCl(f^{-1}(F))$ .Thus we have $NsgCl(f^{-1}(F))=f^{-1}(F)$ which implies that $f^{-1}(F)$ is nano sg-closed in U for every nano closed set F in V. That is, $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nanosg- continuous.

Example: 3.6 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano sg-continuous, then $f(NsgCl(A)$ is not necessarily equal to $NCl(f(A))$ where A $\subseteq$ U.

In Example 3.2, let us define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as f(a) = y, f(b) = x, f(c) = y, f(d) = z. Then f^{-1}(V) = U, $f^{-1}(\phi)$ = ϕ, f^{-1}({y, z}) = {a, c, d}, f^{-1}({x}) = {b}, f^{-1}({x, y, z}) = U. Thus the inverse image of every nano open set in V is nanosg-open in U. Hence $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nanosg– continuous on U.

Let A ={b,d} $\subseteq$ U. Now $NsgCl(A)=\{b,d\}$ and hence $f(NsgCl(A)=f(\{b,d\})=\{x,z\}$. Now $NCl(f(A))=NCl(f(\{b,d\}))=NCl(\{x,z\})=\{x,y,z\}$.That is, the equality does not hold in the above theorem when f is nano continuous and thus $f(NsgCl(A)\neq NCl(f(A))$ even though f is nano sg-continuous.

Theorem: 3.7 Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces where $X\subseteq U$ and $Y\subseteq V$ .Then $\tau_{R'}(Y)=\{V,\phi,L_{R'}(Y),U_{R'}(Y),B_{R'}(Y)\}$ and its basis is given by $B_{R'}=\{V,L_{R'}(Y),B_{R'}(Y)\}$.A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous if and only if the inverse image of every member of $B_{R'}$ is nanosg-open in U.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be nano sg-continuous on $(U,\tau_R(X))$.Let B $\in B_{R'}$. Then B is nano open in $(V,\tau_{R'}(Y))$. Since f is nano sg-continuous, $f^{-1}(B)$ is nano sg-open in U and $f^{-1}(B)\in\tau_R(X)$. Hence the inverse image of every member of $B_{R'}$ is nano sg-open in U.

Conversely, let the inverse image of every member of $B_{R'}$ be nano sg-open in U. Let G be nano open in V. Now $G = \cup\{B : B \in B_1\}$ where $B_1 \subset B_{R'}$. Then $f^{-1}(G) = f^{-1}[\cup\{B : B \in B_1\}] = \cup\{f^{-1}(B) : B \in B_1\}$ where each $f^{-1}(B)$ is nanosg-open in U and their union which is $f^{-1}(G)$ is also nano sg-open in U.
By definition, $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous on $(U, \tau_R(X))$.

Theorem: 3.8 A map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous if and only if $NsgCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset B of V.

Proof: Let $B \subseteq V$ and $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nano sg-continuous. Then $NCl(B)$ is nano closed in $(V, \tau_{R'}(Y))$ and hence $f^{-1}(NCl(B))$ is nano sg-closed in $(U, \tau_R(X))$. Therefore, $NsgCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Since $B \subseteq NCl(B)$, then $f^{-1}(B) \subseteq f^{-1}(NCl(B))$, i.e., $NsgCl(f^{-1}(B)) \subseteq NsgCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Hence $NsgCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$.

Conversely, let $NsgCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset $B \subseteq V$.Now let B benano closed in $(V, \tau_{R'}(Y))$, then $NCl(B) = B$. Given $NsgCl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ .Hence $NsgCl(f^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq NsgCl(f^{-1}(B))$ and hence $NsgCl(f^{-1}(B)) = f^{-1}(B)$ .Thus $f^{-1}(B)$ is nano sg-closed set in $(U, \tau_R(X))$ for every nano closed set B in $(V, \tau_{R'}(Y))$.Hence
$f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous.

The following theorem establishes a criteria fornanosg-continuous functions in terms of inverse image of nano interior of a subset of $(V, \tau_{R'}(Y))$.

Theorem: 3.9 A function $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous if and only if $f^{-1}(NInt(B)) \subseteq NsgInt(f^{-1}(B))$ for every subset B of $(V, \tau_{R'}(Y))$.

Proof: Let $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ be nanosg-continuous and $B \subseteq V$. Then $NInt(B)$ is nano open in V. Now $f^{-1}(NInt(B))$ is nano sg-open in $(U, \tau_R(X))$ i.e., $NsgInt(f^{-1}(NInt(B))) = f^{-1}(NInt(B))$.
Also, for $B \subseteq V$, $NInt(B) \subseteq B$ always. Then $f^{-1}(NInt(B)) \subseteq f^{-1}(B)$.

Therefore, $NsgInt(f^{-1}(NInt(B))) \subseteq NsgInt(f^{-1}(B))$, i.e., $f^{-1}(NInt(B)) \subseteq NsgInt(f^{-1}(B))$.

Conversely, let $f^{-1}(NInt(B)) \subseteq NsgInt(f^{-1}(B))$ for every subset B of V. Let B be nano open in V and hence $NInt(B) = B$. Given $f^{-1}(NInt(B)) \subseteq NsgInt(f^{-1}(B))$, i.e., $f^{-1}(B) \subseteq NsgInt(f^{-1}(B))$. Also $NsgInt(f^{-1}(B)) \subseteq f^{-1}(B)$.Hence $f^{-1}(B) = NsgInt(f^{-1}(B))$ which implies that $f^{-1}(B)$ is nano sg-open in U for every nano open set B of V. Therefore $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano sg-continuous.

Example: 3.10 In Example 3.2, let us define $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ as f(a) = y, f(b) = x, f(c) = z, f(d) = w. Here f is nano sg– continuous since the inverse image of every nano open set in V is nanosg-open in U. Let $B = \{y\} \subset V$. Then $NCl(B) = \{y, z, w\}$. Hence $f^{-1}(NCl(B)) = f^{-1}(\{y, z, w\}) = \{a, c, d\}$.Also $f^{-1}(B) = \{a\}$. Hence $Nsgcl(f^{-1}(B)) = NsgCl(\{a\}) = \{a\}$. Thus $Nsgcl(f^{-1}(B)) \neq f^{-1}(NCl(B)$. Also when $A = \{y, z, w\} \subseteq V$, $f^{-1}(NInt(A)) = f^{-1}(\{y, z\}) = \{a, c\}$. But $NsgInt(f^{-1}(A)) = NsgInt(\{a, c, d\})$ $= \{a, c, d\}$. That is, $f^{-1}(NInt(A)) \neq NsgInt(f^{-1}(A))$. Thus the equality does not hold in Theorems 3.7 and 3.8 when f is nano continuous.

Theorem: 3.11 Let $(U,\tau_R(X))$ and $(V,\tau_{R'}(Y))$ be two Nano topological spaces with respect to $X\subseteq U$ and $Y\subseteq V$ respectively, then for any function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$, the following are equivalent :

1. f is nano sg-continuous
2. The inverse image of every nanoclosed set in V is nano sg- closed in $(U,\tau_R(X))$.
   $f(NsgCl(A))\subseteq NCl(f(A))$ **for every subset A of** $(U,\tau_R(X))$ **.**
3. The inverse image of every member of $B_{R'}$ is nano sg-open in $(U,\tau_R(X))$.
4. $NsgCl(f^{-1}(B))\subseteq f^{-1}(NCl(B))$ **for every subset B of** $(V,\tau_{R'}(Y))$

Proof of the above theorem follows from Theorems 3.3 to 3.8

Theorem: 3.12 Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be an onto,nanosg-continuous function. If A is nano sg-dense in $(U,\tau_R(X))$, then $f(A)$ is nano dense in $(V,\tau_{R'}(Y))$.

Proof: Given A is nano sg-dense in $(U,\tau_R(X))$. Hence $NsgCl(A)=U$. As f is onto , $f(NsgCl(A))=f(U)=V.$ Since f is nano sg-continuous on U, by Theorem 3.5, $f(NsgCl(A))\subseteq NCl(f(A)).$ Hence $V\subseteq NCl(f(A))$. Also $NCl(f(A))\subseteq V$ implies $NCl(f(A))=V$ Hence $f(A)$ is nano dense in $(V,\tau_{R'}(Y))$.Thus a nano continuous function maps nanosg-dense sets into nano dense sets provided it is onto.

Remark: 3.13 We denote the family of all nano sg-open sets in $(U,\tau_R(X))$ by $\tau_R{}^{Nsg}(X).$

Theorem: 3.14 A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous if and only if $f:(U,\tau_R{}^{Nsg}(X))\to(V,\tau_{R'}(Y))$ is nano continuous.

Proof: Assume that $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous. Then $f^{-1}(A)\in\tau_R{}^{Nsg}(X)$ for every $A\in\tau_{R'}(Y)$ .Hence $f:(U,\tau_R{}^{Nsg}(X))\to(V,\tau_{R'}(Y))$ is nano continuous.

Conversely, assume that $f:(U,\tau_R{}^{Nsg}(X))\to(V,\tau_{R'}(Y))$ is nano continuous. Then $f^{-1}(G)\in\tau_R{}^{Nsg}(X)$ for every $G\in\tau_{R'}(Y)$. Then $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is nano sg-continuous.

Remark: 3.15 The composition of two nanosg-continuous maps need not be nanosg-continuous and this is shown by the following example.

Example:3.16 Let $U=V=W$ ={a, b, c, d} with $\tau_R(X)$ ={U, ϕ,{a},{a, b, d},{b, d}}, $\tau_{R'}(Y)$ ={V, ϕ,{b},{a, c},{a, b, c}} and $\tau_{R''}(Z)$ ={W, ϕ, {c}, {a, b}, {a, b, c}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ as f(a) = b, f(b) = c, f(c) = d, f(d) = a and $g:(V,\tau_{R'}(Y))\to(W,\tau_{R''}(Z))$ be the identity map. Then f and g are nano sg– continuous but their composition $g\circ f:(U,\tau_R(X))\to(W,\tau_{R''}(Z))$ is not nano sg– continuous because $F=\{c,d\}$ is nano closed in $(W,\tau_{R''}(Z))$ but $(g\circ f)^{-1}(F)=f^{-1}[g^{-1}(F)]=f^{-1}[g^{-1}(\{c,d\})]=f^{-1}(\{c,d\})=\{b,c\}$ which is not nano sg-closed in $(U,\tau_R(X))$ .Hence the composition of two nanosg-continuous maps need not be nanosg-continuous.

REFERENCES:

1. S.P.Arya, and T.Nour,(1990),Characterizations of s-Normal Spaces, Indian J. Pure Appl.Math.,Vlo.21, Pp. 717-719.
2. P.Bhattacharyya, and B.K.Lahiri, (1987), Semi-Generalized Closed Sets in Topology, Indian J. Pure Appl. Math., Vol. 29, Pp. 375-382.
3. K.Bhuvaneswari, and A.Ezhilarasi, On Nano semi-generalized and Nano generalized–semi closed sets in Nano Topological Spaces, International Journal of Mathematics and Computer Applications Research, (2014), 117-124.

4. M.CaldasCueva, Semi-Generalized Continuous Maps in Topological Spaces, Portugaliae Mathematica, Vol.52 Fasc.4 – 1995.
5. R.Devi, H.Maki, and K.Balachandran, (1993), Semi-Generalized Closed Maps and Generalized-Semi Closed Maps, Mem. Fac. Sci. Kochi Uni., (Maths), Vol.14, Pp. 41-54.
6. GovindappaNavalagi, Properties of gs-closed sets and sg-closed sets in Topology, International Journal of Communication in Topology, Vol.1 Pp. 31-40.
7. M.Lellis Thivagar, and Carmel Richard, On Nano Forms of Weakly Open sets, International Journal Mathematics and Statistics Invention,(2013),31-37.
8. M.LellisThivagar, and Carmel Richard, On Nano Continuity, Mathematical Theory and Modeling, (2013), no.7, 32-37.
9. N.Levine, (1970), Generalized closed sets in Topological Spaces, Rend. Circ. Mat. Palermo, Vol 19, Pp.89-96.
10. P.Sundaram,H.Maki, and K.Balachandran, (1991), Semi-Generalized Continuous Maps and Semi-$T_{1/2}$ Spaces, Bull. Fukuoka Univ.Ed.Part-III, Vol.40, Pp.33-40.

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