ON IDENTIFICATION OF THE NATURE OF TRIANGLE BY NEW APPROACH

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ABSTRACT

In this paper, we introduced new technique to explain nature of the triangle. We established necessary procedure to identify right, acute and obtuse angled triangle.

Key words: Pythagoras Theorem, Acute angled triangle, Obtuse angled triangle.

AMS Subject Classifications: 51K, 52B.

INTRODUCTION

Triangle, in Mathematics, plane figure bounded by three straight lines, the sides, which intersect at three points called the vertices. Any one of the sides may be considered the base of the triangle. The perpendicular distance from a base to the opposite vertex is called altitude. The line segment joining the midpoint of a side to the opposite vertex is called Median.

The theorem about right triangles is attributed to and now bears the name a Greek, Pythagoras of Samos, born around 570-540 B.C. Pythagoras is often credited with the first proof of the theorem, however his actual written proof has not been found. Earlier civilizations definitely knew about this geometric fact and perhaps after his travel, Pythagoras took this information back to Greek. It had been discovered by Boudhaayan, an Indian mathematician, three centuries before Pythagoras [9]

The Babylonians discovered the triples (sets of three numbers that satisfy the Pythagoras Theorem) much earlier, approximately 1900-1600 BC, long before Pythagoras time. The Chinese may have proven the Pythagoras Theorem earliest; some estimates as early as 1100 B.C., although 6th Century B.C. is more generally accepted [5]. The Chinese grasped many right-angled triangle principles early on, and applied them to practical problems.

PRELIMINARY

In Euclidean geometry, Cosine function is strictly decreasing on [0, π]. The sum of the degree measures of the interior angles of a triangle is 180. The degree measure of an interior angle of a triangle is between 0 and 180. That means if α is the degree measure of an interior angle of a triangle then 0 < α < 180.

Cosine function and its graph

Sketch for the Graph of \( f(x) = \cos x \) for \( 0 \leq x \leq \pi \), is given below;

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As we see from the graph \( \cos 0 = 1, \cos \frac{\pi}{2} = 0, \cos \pi = -1 \). Thus if \( 0 < \alpha < \frac{\pi}{2} \), then \( \cos 0 > \cos \alpha > \cos \frac{\pi}{2} \Rightarrow 1 > \cos \alpha > 0 \). If \( \frac{\pi}{2} < \alpha < \pi \), then \( \cos \frac{\pi}{2} > \cos \alpha > \cos \pi \Rightarrow 0 > \cos \alpha > -1 \).

**MAIN THEOREM**

Our objective is to find out necessary conditions, which enable to identify a triangle, whether it is right angled triangle, acute angled triangle or obtuse angled triangle.

**Derivation of Main Theorem**

Let us consider a triangle \( \triangle ABC \) as given below,

![Triangle Diagram](image)

\( b = b_1 + b_2; \ b_1 \neq b_2 \).

Let us assume that 'a' is a longest side in the \( \triangle ABC \). Let 'd' be the length of the unique perpendicular segment from B to AC. From above \( \triangle ABC \), we have the following conditions.

\[ \alpha \geq \beta, \alpha \geq \gamma; \ b_1 = c \cos \alpha; \ d = c \sin \alpha; \ AC = b = (b_1 + b_2); \ b_2 = (b - b_1). \]

Now from \( \triangle BDC \), since line BD is perpendicular on the line AC, hence we have the following:

\[ d^2 + b_2^2 = a^2 \Rightarrow b_2^2 = a^2 - d^2 \Rightarrow b_2^2 = a^2 - (c \sin \alpha)^2 \Rightarrow b_2^2 = a^2 - c^2 \sin^2 \alpha \]

\[ \Rightarrow (b - b_1)^2 = a^2 - c^2 \sin^2 \alpha \Rightarrow (b - c \cos \alpha)^2 = a^2 - c^2 \sin^2 \alpha \]

\[ \Rightarrow b^2 - 2bc \cos \alpha + c^2 \cos^2 \alpha = a^2 - c^2 \sin^2 \alpha \]

\[ \Rightarrow b^2 - 2bc \cos \alpha + c^2 (\sin^2 \alpha + \cos^2 \alpha) = a^2 \]

\[ \Rightarrow b^2 = a^2 + c^2 - 2bc \cos \alpha \]

\[ \Rightarrow a^2 = b^2 + c^2 - 2bc \cos \alpha \]
From equation (1), we have following conditions;

[I]. Let us assume that
\[ a^2 = b^2 + c^2 \implies b^2 + c^2 - 2bc\cos\alpha = b^2 + c^2 \implies -2bc\cos\alpha = 0 \implies 2bc\cos\alpha = 0 \]
\[ \cos\alpha = 0, \text{ (since } 2 \neq 0, b \neq 0, c \neq 0 \implies \alpha = 90, \text{ } \alpha \text{ is right angle.} \]
Therefore \( \triangle ABC \) is right angled triangle.

[II]. Let us assume that
\[ a^2 < (b^2 + c^2) \implies (b^2 + c^2 - 2bc\cos\alpha) < (b^2 + c^2) \implies -2bc\cos\alpha < 0 \implies 2bc\cos\alpha > 0 \]
\[ \cos\alpha > 0, \text{ (since } C \neq 0, b \neq 0, c \neq 0) \implies \alpha < 90, \text{ because Cosine function is decreasing on } [0, \pi], \text{ hence } \alpha \text{ is acute angle.} \]
Therefore \( \triangle ABC \) is acute angled triangle.

[III]. Again, let us assume that
\[ a^2 > (b^2 + c^2) \implies (b^2 + c^2 - 2bc\cos\alpha) > (b^2 + c^2) \implies -2bc\cos\alpha > 0 \implies 2bc\cos\alpha < 0 \implies \cos\alpha < 0 \]
\[ \text{(since } 2 \neq 0, b \neq 0, c \neq 0) \implies \alpha > 90, \text{ because Cosine function is decreasing on } [0, \pi], \text{ hence } \alpha \text{ is obtuse angle.} \]
Therefore \( \triangle ABC \) is obtuse angled triangle.

Results: Thus we have following conditions from the theorem;
[i]. If \( a^2 = b^2 + c^2 \), then \( \triangle ABC \) is right angled triangle.
[ii]. If, \( a^2 < (b^2 + c^2) \), then \( \triangle ABC \) is acute angled triangle.
[iii]. If, \( a^2 > (b^2 + c^2) \), then \( \triangle ABC \) is obtuse angled triangle.

SIGNIFICANCE OF THE THEOREM

Triangle is the simplest polygon; any polygon can be decomposed into triangles. Thus the properties of a Polygon can be studied based on properties of a triangle. Among similar properties of a triangle the one which save a time is preferable. This triangle formula saves time.

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