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THE NUMBER OF MINIMUM CO – ISOLATED LOCATING DOMINATING SETS OF CYCLES

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ABSTRACT

Let G (V, E) be a simple, finite, undirected connected graph. A non – empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in V – S is adjacent to atleast one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices v, w $\in$ V – S, $N(v) \cap S \neq N(w) \cap S$. A locating dominating set $S \subseteq V$ is called a co – isolated locating dominating set, if there exists atleast one isolated vertex in <V – S >. The co – isolated locating domination number γ_{cild} is the minimum cardinality of a co – isolated locating dominating set. γ_{Dcild} is the number of minimum co – isolated locating dominating set of a graph G. In this paper, the number γ_{Dcild} is obtained for a cycle C_n, $n \geq 3$.

Keywords: *Dominating set, locating dominating set, co – isolated locating dominating set, co – isolated locating dominating number.*

1. INTRODUCTION

Let G = (V, E) be a simple graph of order n. For v ∈ V(G), the neighborhood $N_G(v)$ (or simply N(v)) of v is the set of all vertices adjacent to v in G. The concept of domination in graphs was introduced by Ore [7]. A non – empty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in V(G) – S is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D. F. Rall and P. J. Slater in [8]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices v, w ∈ V(G) – S, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct . The location dominating number of G is defined as the minimum number of vertices in a locating dominating set in G. A locating dominating set $S \subseteq V(G)$ is called a co–isolated locating dominating set , if <V – S> contains atleast one isolated vertex. The minimum cardinality of a co – isolated locating dominating set is called the co – isolated locating domination number $\gamma_{cild}(G)$. $\gamma_{Dcild}(G)$ is the number of minimum co – isolated locating dominating sets of a graph G. It is well known that the concept of domination is originated from the game of chess board. The problem of finding the minimum number of stones is another aspect and the number of ways of placing the minimum number of stones is another aspect. In this paper, the second aspect of the problem, that is, γ_{Dcild} is obtained for cycles C_n, $n \geq 4$.

2. PRIOR RESULTS

The following results are obtained in [3] & [4]

Theorem 2.1[3]: For every non – trivial simple connected graph G, $1 \leq \gamma_{cild}(G) \leq n-1$.

Theorem 2.2[3]: $\gamma_{cild}(G) = 1$ if and only if $G \cong K_2$.

Theorem 2.3 [3]: $\gamma_{cild}(K_n) = n – 1$, where K_n is a complete graph on n vertices.

Theorem 2.4 [3]: $\gamma_{cild}(K_n – e) = n – 1$, $e \in E(K_n)$

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Observation 2.5 [4]: If S is an co – isolated locating dominating set of G(V, E) with | S | = k, then V(G) – S contains atmost $nC_1 + nC_2 + \ldots + nC_k$ vertices.

Theorem 2.6 [4]: If P_n is a path on n vertices, n ≥ 3

$$\gamma_{cild}(P_n) = \begin{cases} 2\left\lfloor \frac{n}{5} \right\rfloor & n \equiv 0 \ (mod\ 5) \\ 2\left\lfloor \frac{n}{5} \right\rfloor + 1; & n \equiv 1 \ or\ 2\ (mod\ 5) \\ 2\left\lfloor \frac{n}{5} \right\rfloor + 2\,; & n \equiv 3 \ or\ 4\ (mod\ 5) \end{cases}$$

3. MAIN RESULTS

In the following, co-isolated locating domination number $\gamma_{cild}(C_n)$ of a cycle is found.

Theorem 3.1: If C_n (n ≥ 3) is a cycle on n vertices, then $\gamma_{cild}(C_n) = \left\lceil \frac{2n}{5} \right\rceil$.

Proof: Let $V(C_n) = \{v_1, v_2, v_3, \ldots, v_{n-1}, v_n\}$, where $(v_i, v_{i+1}), (v_n, v_1) \in E(C_n)$, i = 1, 2, …, n-1.

Case-(i): n ≡ 0 (mod 5)

Let n = 5k, k ≥ 1. The theorem is proved by induction on k.

For k = 1, the set $S_1 = \{v_2, v_4\}$ is a minimum co – isolated locating dominating set of C_5 and hence $\gamma_{cild}(C_5) = 2$. For k = 2, the set $S_2 = S_1 \cup \{v_7, v_9\}$ is a minimum co – isolated locating dominating set of C_{10} and hence
$\gamma_{cild}(C_{10}) = 4 = \gamma_{cild}(C_5) + 2 = 2k$, where k = 2.

Assume that the theorem holds for k = j – 1.

That is, for the cycle $C_{5(j-1)}$, $\gamma_{cild}(C_{5(j-1)}) = \gamma_{cild}(C_{5(j-2)}) + 2 = 2(j-1)$. The set $S_{j-1} = S_{j-2} \cup \{v_{5j-8}, v_{5j-6}\}$ is a minimum co – isolated locating dominating set of $C_{5(j-1)}$. Let k = j.

Clearly, $S_j = S_{j-1} \cup \{v_{5j-3}, v_{5j-1}\}$ is a minimum co – isolated locating dominating set of C_{5j} and hence $\gamma_{cild}(C_j) = 2j$. Therefore, the theorem is proved for k =j. By induction hypothesis, $\gamma_{cild}(C_n) = \gamma_{cild}(C_{5k}) = 2k = \left\lceil \frac{2n}{5} \right\rceil$, for all n ≥ 3 and the set

$S = \bigcup_{j=1}^{k} \{v_{5j-3}, v_{5j-1}\}$ is a minimum co-isolated dominating set of C_n, where n = 5k.

Case-(ii): n ≡ 1 (mod 5)

Let n = 5k + 1, k ≥ 1. Any set S of 2k vertices of C_{5k+1} is not a co – isolated locating dominating set of C_{5k+1}, since either S is not a locating set of C_{5k+1} or S contains any isolated vertex. Therefore, a minimum co – isolated locating dominating set of C_{5k+1} contains 2k + 1 vertices. Then, $S_1 = S \cup \{v_{5k+1}\}$ is a minimum co – isolated locating dominating set of C_{5k+1}. Therefore, $\gamma_{cild}(C_{5k+2}) = 2k + 1$.

Case-(iii): n ≡ 2 (mod 5)

Let n = 5k + 2. Then, $S_2 = S_1 \cup \{v_{5k+2}\}$ is a minimum co – isolated locating dominating set of C_{5k+2}. Therefore,
$\gamma_{cild}(C_{5k+2}) = 2k + 1$.

Case-(iv): n ≡ 3 (mod 5)

Let n = 5k + 3. Then, $S_3 = S_1 \cup \{v_{5k+2}, v_{5k+3}\}$ is a minimum co – isolated locating dominating set of C_{5k+3}. Therefore,
$\gamma_{cild}(C_{5k+3}) = 2k + 2$.

Case-(iv): n ≡ 4 (mod 5)

Let n = 5k + 4. Then, $S_4 = S_1 \cup \{v_{5k+2}, v_{5k+4}\}$ is a minimum co – isolated locating dominating set of C_{5k+4}. Therefore,
$\gamma_{cild}(C_{5k+4}) = 2k + 2$.

In the following, the number γ_{Dcild} (C_{5k}) of minimum co – isolated locating dominating sets of C_{5k} is found.

Theorem 3.2: γ_{Dcild} (C_{5k}) = 4, where k ≥ 1.

Proof: The Theorem is proved by the method of induction on k.

Let $V(C_{5k}) = \{v_1, v_2, v_3, \ldots, v_{5k-2}, v_{5k-1}, v_{5k}\}$. For k = 1, the four sets $D_1^1 = \{v_2, v_4\}$; $D_2^1 = \{v_1, v_3\}$; $D_3^1 = \{v_1, v_4\}$ and $D_4^1 = \{v_2, v_5\}$ are the minimum co – isolated locating dominating sets of C_5.

Therefore, γ_{Dcild} (C_5) = 4. For k = 2, the four sets $D_1^2 = D_1^1 \cup \{v_7, v_9\}$; $D_2^2 = D_2^1 \cup \{v_6, v_8\}$; $D_3^2 = D_3^1 \cup \{v_6, v_9\}$ and $D_4^2 = D_4^1 \cup \{v_7, v_{10}\}$ are the minimum co – isolated locating dominating sets of C_{10} and hence γ_{Dcild} (C_{10}) = 4. Assume that the theorem holds for k = j – 1. That is, γ_{Dcild} ($C_{5(j-1)}$) = 4.

Then, $D_1^{j-1} = D_1^{j-2} \cup \{v_{5j-8}, v_{5j-6}\}$; $D_2^{j-1} = D_2^{j-2} \cup \{v_{5j-9}, v_{5j-7}\}$; $D_3^{j-1} = D_3^{j-2} \cup \{v_{5j-9}, v_{5j-6}\}$ and $D_4^{j-1} = D_4^{j-2} \cup \{v_{5j-8}, v_{5j-5}\}$ are the only minimum co – isolated locating dominating sets of $C_{5(j-1)}$, since $| D_i^{j-1} | = 2j - 2 = \gamma_{\text{cild}} (C_{5(j-1)})$; i = 1, 2, 3, 4. Let k = j. Again, the sets $D_1^j = D_1^{j-1} \cup \{v_{5j-3}, v_{5j-1}\}$; $D_2^j = D_2^{j-1} \cup \{v_{5j-4}, v_{5j-2}\}$; $D_3^j = D_3^{j-1} \cup \{v_{5j-4}, v_{5j-1}\}$and $D_4^j = D_4^{j-1} \cup \{v_{5j-3}, v_{5j}\}$ are the only minimum co – isolated locating dominating sets of C_{5j} , since $| D_i^j | = 2j = \gamma_{\text{cild}} (C_{5j})$; i = 1, 2, 3, 4. Therefore, the theorem is proved for k = j. By induction hypothesis, γ_{Dcild} (C_{5k}) = 4; k ≥ 1.

Theorem 3.3: γ_{Dcild} (C_{5k+2}) = 4, where k ≥ 1.

Proof: By Theorem 3.2, γ_{cild} (C_{5k}) = 4 and the corresponding four minimum co–isolated locating dominating sets of C_{5k} are given by $D_1=\bigcup_{j=1}^{k}\{ v_{5j-3}, v_{5j-1}\}$; $D_2=\bigcup_{j=1}^{k}\{ v_{5j-4}, v_{5j-2}\}$; $D_3 = \bigcup_{j=1}^{k}\{ v_{5j-4}, v_{5j-1}\}$ and $D_4 = \bigcup_{j=1}^{k}\{ v_{5j-3}, v_{5j}\}$.

Let $D_1' = D_1 \cup \{v_{5j+2}\}$; $D_2' = D_2 \cup \{v_{5j+1}\}$; $D_3' = D_3 \cup \{v_{5j+1}\}$ and $D_4' = D_4 \cup \{v_{5j+2}\}$.

The sets D_1', D_2', D_3' and D_4' form a minimum co – isolated locating dominating sets of $C_{5k+2.}$

Also, these are minimum co – isolated locating dominating sets of C_{5k+2}, since
$| D_i'| = | D_i| + 1 = 2k + 1 = \gamma_{\text{cild}} (C_{5k+2})$; i = 1, 2, 3, 4. Hence, γ_{Dcild} (C_{5k+2}) = 4.

Theorem 3.4: γ_{Dcild} (C_{5k+4}) = 4, where k ≥ 1.

Proof: By Theorem 3.2., γ_{Dcild} (C_{5k}) = 4. The corresponding four minimum co – isolated locating dominating sets of C_{5k} are given by $D_1 = \bigcup_{j=1}^{k}\{ v_{5j-3}, v_{5j-1}\}$;
$D_2 = \bigcup_{j=1}^{k}\{ v_{5j-4}, v_{5j-2}\}$;
$D_3 = \bigcup_{j=1}^{k}\{ v_{5j-4}, v_{5j-1}\}$ and $D_4 = \bigcup_{j=1}^{k}\{ v_{5j-3}, v_{5j}\}$. Let
$D_1' = D_1 \cup \{v_{5j+2}, v_{5j+4}\}$; $D_2' = D_2 \cup \{v_{5j+1}, v_{5j+3}\}$; $D_3' = D_3 \cup \{v_{5j+1}, v_{5j+3}\}$ and
$D_4' = D_4 \cup \{v_{5j+2}, v_{5j+4}\}$.

The sets D_1', D_2', D_3' and D_4' form a minimum co–isolated locating dominating sets of C_{5k+4}.

Also, these are minimum co – isolated locating dominating sets of C_{5k+2}, since$| D_i'| = | D_i| + 1 = 2k + 1 = \gamma_{\text{cild}} (C_{5k+2})$; i = 1, 2, 3, 4. Hence, γ_{Dcild} (C_{5k+2}) = 4.

Theorem 3.5: γ_{Dcild} (C_{5k+1}) = 10k – 2, where k ≥ 1.

Proof: Let $V(C_{5k+1}) = \{v_1, v_2, v_3, \ldots, v_{5k}, v_{5k+1}\}$. Any co – isolated locating dominating set D of C_{5k+1} contains either (i) two adjacent vertices or (ii) three vertices v_i, v_j,v_k such that $d(v_i, v_j) = d(v_j, v_k) = 2$.

Case-(i): D contains two adjacent vertices

Then, the number of minimum co – isolated locating dominating sets is 5k + 1. This is proved by the method of induction on k. For k = 1, the co – isolated locating dominating sets of C_6 containing two adjacent vertices are given by, $D_1 = \{v_1, v_2, v_5\}$; $D_2 = \{v_2, v_3, v_6\}$; $D_3 = \{v_3, v_4, v_1\}$; $D_4 = \{v_4, v_5, v_2\}$; $D_5 = \{v_5, v_6, v_3\}$ and $D_6 = \{v_6, v_1, v_4\}$.

That is, $D_i = \{v_i, v_{i+1}, v_{i+4}\}$; i = 1, 2, 3, …, 6 and the addition is taken over modulo 6. Therefore, the number of minimum co – isolated locating dominating sets containing a pair of adjacent vertices is 6 = 5k + 1, when k = 1. For k = 2, the co – isolated locating dominating sets containing a pair of adjacent vertices is given by, $D_i = \{v_i, v_{i+1}, v_{i+3}, v_{i+6}, v_{i+9}\}$; i = 1, 2, …, 11 and the addition is taken over modulo 11. These sets are also minimum co – isolated locating dominating sets, since $|D_i| = 5 = \gamma_{cild}(C_{11})$. Hence, $\gamma_{Dcild}(C_{11}) = 11 = 5k + 1$, when k = 2. Assume that the result holds for k = j – 1. That is, the number of minimum co – isolated locating dominating sets containing a pair of adjacent vertices in $C_{5(j-1)+1}$ is 5(j - 1)+ 1 = 5j – 4 and the minimum co-isolated locating dominating sets are given by $D_i = \{v_i, v_{i+1}, v_{i+3}, v_{i+6}, v_{i+9}, \ldots, v_{i+(5j-5)}, v_{i+(5j-2)}\}$; i = 1, 2, …, 5j – 4 and the addition is taken over modulo 5j – 4. Let k = j. The minimum co – isolated locating dominating sets containing a pair of adjacent vertices in C_{5j+1} is given by $D_i = \{v_i, v_{i+1}, v_{i+3}, v_{i+6}, v_{i+9}, \ldots, v_{i+(5j-4)}, v_{i+(5j-1)}\}$; i = 1, 2, …., 5j + 1 and the addition is taken over modulo 5j+1. By induction hypothesis, the number of minimum co – isolated locating dominating sets of C_{5k+1} is 5k + 1

Case-(ii): D contains three vertices v_i, v_j, v_k such that $d(v_i, v_j) = d(v_j, v_k) = 2$.

The number of minimum co – isolated locating dominating sets in this case is 5k – 3, This result is proved by the method of induction on k. For k = 1, the co – isolated locating dominating sets of C_6 are given by $D_1 = \{v_1, v_3, v_5\}$; $D_2 = \{v_2, v_4, v_6\}$. That is, $D_i = \{v_i, v_{i+2}, v_{i+4}\}$; i = 1, 2 and the addition is taken over modulo 6. Therefore, the number of minimum co – isolated locating dominating sets is 2 = 5k – 3, when k = 1. For k = 2, the co – isolated locating dominating sets are given by $D_i = \{v_i, v_{i+2}, v_{i+4}, v_{i+7}, v_{i+9}\}$; i = 1, 2, …, 7 and the addition is taken over modulo 11. These sets are also minimum co – isolated locating dominating sets, since $|D_i| = 5 = \gamma_{cild}(C_{11})$. Hence, $\gamma_{Dcild}(C_{11}) = 7 = 5k - 3$, when k = 2. Assume that the result holds for k = j-1. That is, the number of minimum co – isolated locating dominating sets of $C_{5(k-1)+1}$ is 5j – 8 and the minimum co-isolated dominating sets are given by $D_i = \{v_i, v_{i+2}, v_{i+4}, v_{i+7}, v_{i+9}, \ldots, v_{i+(5j-8)}, v_{i+(5j-6)}\}$; i = 1, 2, …, 5j – 8 and the addition is taken over modulo 5j-8.

Let k =j. The minimum co – isolated locating dominating sets of C_{5j+1} are given by
$D_i = \{v_i, v_{i+2}, v_{i+4}, v_{i+7}, v_{i+9}, \ldots, v_{i+(5j-3)}, v_{i+(5j-1)}\}$; i = 1, 2, …, 5j - 3, and the addition is taken over modulo 5j - 3. By induction hypothesis, the number of minimum co – isolated locating dominating sets of C_{5k+1} is 5k - 3, for all k, k ≥ 1.

There is no other co – isolated locating dominating sets of C_{5k+1} having 2k + 1 vertices. By Case(i) and Case(ii), the number of minimum co – isolated locating dominating sets of C_{5k+1} is 5k + 1 + 5k – 3 = 10k – 2. Hence, $\gamma_{Dcild}(C_{5k+1}) = 10k - 2$.

Theorem 3.6: $\gamma_{Dcild}(C_{5k+3}) = 10k - 2$; k ≥ 1.

Proof: By Theorem 3.5, $\gamma_{Dcild}(C_{5k+1}) = 10k - 2$ and let the corresponding minimum co – isolated locating dominating sets of C_{5k+1} be D_i ; i = 1, 2, …, 10k – 2.

Let $D_i' = D_i \cup \{v_{i+(5k+2)}\}$; i = 1, 2… 10k – 2. Then the sets D_i' will form minimum co – isolated locating dominating sets of C_{5k+3}. Also, these sets are minimum co – isolated locating dominating sets of C_{5k+3}, since $|D_i'| = |D_i| + 1 = 2k + 1 + 1 = 2k + 2 = \gamma_{cild}(C_{5k+3})$; i = 1, 2, 3, …, 10k – 2. Hence, $\gamma_{Dcild}(C_{5k+3}) = 10k - 2$.

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