International Research Journal of Pure Algebra-4(10), 2014, 599-602
Available online through www.rjpa.info ISSN 2248–9037

ON SEMI-MAXIMAL WEAKLY OPEN AND SEMI-MINIMAL WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

Vivekananda Dembre*
Department of Mathematics,
Rani Channamma University, Belagavi-591156, Karnataka, India.

Jeetendra Gurjar
Department of Mathematics,
Shaikh College of Engineering and Diploma , Belagavi-591156, Karnataka, India.

(Received On: 15-10-14; Revised & Accepted On: 27-10-14)

ABSTRACT

In this paper new class of sets called semi- maximal weakly open sets and semi-minimal weakly closed sets are introduced in topological spaces. We show that the complement of semi-maximal weakly open set is a semi-minimal weakly closed set and some properties of the new concepts have been studied.

Keywords: *Maximal open set, Minimal closed set, Maximal weakly open set, Minimal weakly closed set, Semi- Maximal weakly open set, Semi- Minimal weakly closed set.*

Mathematics subject classification (2000): *54A05.*

1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)$[$M_aC(X)$].The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 1963, N.Levine[4] introduced and studied semi-open sets. A subset A of a topological space X is said to be semi-open set if there exist some open set U such that $U\subset A\subset Cl(U)$. The family of all semi-open sets of X is denoted by SO(X). The Complement[5] of semi-open set is called semi-closed set in X. The family of all semi-closed sets are denoted by SC(X). In the year 2009, S.S.Benchalli and B.M.Ittanagi [6] introduced and studied semi-maximal open and semi-minimal closed sets in topological spaces. In the year 2000, M.Sheik john [7] introduced and studied weakly closed sets and weakly open sets in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre[8] introduced and studied maximal weakly open sets and minimal weakly closed sets in topological spaces.

Definition 1.1[1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

Definition 1.2 [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

Definition 1.3[3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

Definition 1.4 [3]:]: A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

Definition 1.5 [4]: A subset A of a topological spaces X is said to be semi-open set if there exist some open set U such that $U\subset A\subset Cl(U)$.

Corresponding Author: Vivekananda Dembre

Definition 1.6 [5]: The complement of semi-open set is called semi-closed set in X.

Definition 1.7 [6]: A set A in a topological space X is said to be semi maximal open set if there exists a maximal open set M such that $M \subset A \subset Cl(M)$.

Definition 1.8 [6]: A subset N of a topological space X is said to be semi-minimal closed set if X-N is semi-maximal open set.

Definition 1.9 [7]: A subset A of (X, τ) is called weakly closed set if $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is Semi-open in X.

Definition1.10[7]: A subset A in (X, τ) is called weakly open set in X if A^c is weakly closed set in X.

Definition1.11[8]: A proper non-empty weakly open subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

Definition1.12[8]: A proper non-empty weakly closed subset F of X is said to be minimal weakly closed set if any weakly closed set which is contained in F is φ or F.

2. SEMI MAXIMAL WEAKLY OPEN SETS

Definition 2.1: A set A in a topological space X is said to be semi-maximal weakly open set if there exists a maximal weakly open set M Such that $M \subset A \subset S\text{-}Cl(M)$.

The family of all semi-maximal weakly open sets in a topological space X is denoted by $SM_awo(X)$.

Example 2.2: Let X={a,b,c}, τ ={ X, φ ,{a},{b},{a,b}} be a topological space.

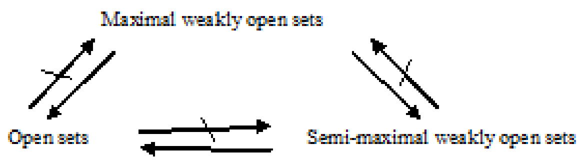
Weakly open sets: {X, φ,{a},{b},{a,b}}

Maximal weakly open sets are : {a,b}

Semi-maximal-weakly-open-sets: {X, {a,b}}

$M_awo(x) \subset SM_awo(x)$.

The above results are given in below implication diagram.



Theorem 2.3: If M is a semi- maximal weakly open set in a topological space X and $M \subset N \subset S\text{-}Cl(M)$ then N is also semi-maximal weakly open in X.

Proof: Let M be a semi-maximal weakly open in X. Then by definition 2.1 there exists a maximal weakly open set U in X such that $U \subset M \subset S\text{-}Cl(U)$. Since $M \subset S\text{-}Cl(U)$ it follows that $S\text{-}Cl(M) \subset Cl(S\text{-}Cl(U)) = S\text{-}Cl(U)$. But from hypothesis $N \subset S\text{-}Cl(M)$ therefore it follows that $U \subset N \subset S\text{-}Cl(U)$. Thus there exists a maximal weakly open set U such that $U \subset N \subset S\text{-}Cl(U)$. Therefore by definition 2.1 it follows that N is semi-maximal weakly open in X.

Theorem 2.4: Let X be a topological space and $M_iwo(x)$ be the class of all maximal weakly open sets in X the following results hold good.
(i) $M_awo(x) \subset SM_awo(X)$
(ii) If $M \in SM_awo(X)$ and $M \subset N \subset S\text{-}Cl(M)$ then $N \subset SM_awo(X)$.

Proof: This follows from theorem 2.3.

Theorem 2.5: : Let X be a topological space. Y be subspace of X and M be a subset of Y. If M is semi-maximal weakly open in X then M is semi-maximal weakly open in Y.

Proof: Suppose M is semi-maximal weakly open in X. By definition 2.1 there exists a maximal weakly open set N in X such that $N \subset M \subset S\text{-}Cl(N)$. Now $N \subset M \subset Y$. Hence $Y \cap N = N$. Since N is maximal weakly open in X. $Y \cap N = N$ is maximal weakly open in Y. Now we have $N \subset M \subset S\text{-}Cl(N)$.Therefore $Y \cap N \subset Y \cap M \subset Y \cap S\text{-}Cl(N)$, which implies $N \subset M \subset S\text{-}Cl\,\gamma(N)$. Thus there exists a maximal weakly open set N in Y Such that $N \subset M \subset S\text{ -}Cl\gamma(N)$. Therefore by definition 2.1 it follows that M is semi-maximal weakly open in Y.

Theorem 2.6: Let X be a topological space. Let M,N be maximal weakly open sets in X and $U \subset X$ such that $N \subset U \subset S\text{-}Cl(N)$ if $M \cap N = \emptyset$ then $U \cap W = \emptyset$.

Proof: Since $M \cap N = \emptyset$, it follows that $N \subset X - M$ therefore $S\text{-}Cl(N) \subset Cl(X\text{-}M) = X\text{-}M$. Since X-M is minimal weakly closed set and every minimal weakly closed set is closed set . Also we have $N \subset U \subset S\text{-}Cl(N)$. Therefore $U \subset S\text{ -}Cl(N) \subset X\text{-}M$ .Thus $U \subset X\text{-}M$ which means $U \cap W = \emptyset$.

Theorem 2.7: Intersection of two semi-minimal weakly open sets need not be semi-minimal weakly open . It can be Shown by the following example.

LetX={a,b,c,d}, τ ={X, φ, {a,b},{c,d}} be a topological space.

Semi-maximal-weakly-open-sets:{X,{a,b,c},{a,b,d},{b,c,d},{a,c,d}} take any two semi-maximal weakly open sets {a,b,d} ∩ {b,c,d}={b,d}Which is not a semi-maximal weakly open set.

3. SEMI-MINIMAL WEAKLY CLOSED SETS

Definition 3.1: A subset N of a topological space X is said to be semi-minimal weakly closed set if X-N is semi-maximal weakly open set.

The family of all semi-minimal weakly closed sets in a topological space X is denoted by $Sm_iC(X)$.

Example 3.2: Let X= {a,b,c}, τ = {X, φ,{a},{b},{a,b}} be a topological space.

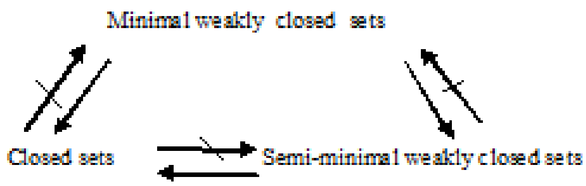
Closed sets are: {X, φ ,{b,c},{a,c}{c}}

Weakly closed sets: {X, φ,{b,c},{a,c},{c}.

Minimal weakly Closed sets: {c}

Semi-minimal-weakly-closed-sets: {φ,{c}}

The above results are given in below implication diagram.



Theorem 3.3: A subset W of a topological space X is semi-minimal weakly closed iff there exists a minimal weakly closed set N in X such that $int(N) \subset W \subset N$.

Proof: Suppose W is a semi-minimal weakly closed in X then by definition 3.1 X-W is semi-maximal weakly open in X. Therefore by definition 2.1 there exists a maximal weakly open set M such that $M \subset X\text{-}W \subset S\text{-}Cl(M)$ which implies that $X\text{-}[S\text{-}Cl(M)] \subset X\text{-}[X\text{-}W] \subset X\text{-}M$ which implies $X\text{-}[S\text{-}Cl(M)] \subset W \subset X\text{-}M$. But it is know that $X\text{-}[S\text{-}Cl(M)] = int(X\text{-}M)$ take $X\text{-}M = N$ so,that N is a minimal weakly closed set such that $int(N) \subset W \subset N$.

Conversly, suppose that there exist a minimal weakly closed set N in X such that int(N) $\subset$W $\subset$N. Therefore it follows that X-N $\subset$[X-W]$\subset$X-int(N).But it is know that X-int(N))=Cl(X-N). Therefore there exists a maximal weakly open set X-N such that X-N $\subset$X-W$\subset$S-Cl(X-N). Thus by definition 2.1 it follows that X-W is semi-maximal weakly open in X. Hence by definition 3.1 it follows that W is Semi-minimal weakly closed set.

Theorem 3.4: If N is semi-minimal weakly closed in X and int(N)$\subset$W$\subset$N then W is semi-minimal weakly closed in X.

Proof: Let N be semi-minimal weakly closed in X then by definition of semi-minimal weakly closed sets there exists a minimal weakly closed set F such that int(F) $\subset$N $\subset$F. Now int(F) $\subset$N which implies int(F)=int(int(F))$\subset$int(N). But int(N) $\subset$W, we have int(F) $\subset$W. Further since int(F) $\subset$ int(N) $\subset$W $\subset$ N $\subset$F. It follows that int(F) $\subset$W $\subset$ F. Thus there exists a minimal weakly closed set F such that int(F) $\subset$W $\subset$F therefore W is semi-minimal weakly closed in X.

Theorem 3.5: The following three properties of a subset N of a topological space X are equivalent.
(i) N is semi-minimal weakly closed set in X
(ii) int(cl(N)) $\subset$N
(iii) (X-N) is semi-maximal weakly open set in X.

REFERENCES

1. F.Nakaoka and F.Oda, Some Application of Maximal open sets, Int.J.Math.Math.sci.Vol 27, No.8, 471-476 (2001).
2. F.Nakaoka and F.Oda, Some Properties of Maximal open sets, Int.J.Math.Math.sci.Vol 21, No.21, 1331-1340 (2003).
3. F.Nakaoka and F.Oda, on Minimal closed sets, Proceeding of topological spaces and it's application, 5, 19-21 (2003).
4. N.Levine (1963), Semi-open sets and Semi-continuity in topological spaces Amer,Math, Monthly 70,36-41.
5. N.Biswas (1970), on Characterization of Semi- Continuous function, Attiaccad, Naz.Linceirend.Cl.sci.Fis. Mat.Natur, 8(48) 339-462.
6. S.S.Benchalli and Basavaraj ittannagi, Semi-maximal open and Semi-minimal closed sets in topological spaces, International Journal of Mathematics and Computing Applications Vol -1, No 1-2 December -January 2009,pp 23-27.
7. M.Shiek John, A study on generalizations of closed sets on continuous maps in topological and bitopological spaces, ph.d thesis bharathiar university, Ciombatore.(2002).
8. R.S.Wali and Vivekananda Dembre, Minimal Weakly Closed sets and Maximal Weakly Open Sets in Topological Spaces, Intenational Research Journal of Pure Algebra – Vol- 4(9), Sept-2014.

Source of Support: Nil, Conflict of interest: None Declared