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SOME NEW RESULTS IN TOPOLOGICAL SPACE FOR NON–SYMMETRIC RATIONAL EXPRESSION CONCERNING 2-BANACH SPACE

[1]Ramakant Bhardwaj, [2]B .R. Wadkar* and [3]Rajesh Shrivastava

[1]Deptt. of Mathematics, "Truba Institute of Engg. and I.T.", Bhopal (M.P) and Research Guide for Mathematics, "Singhania University" Pacheri Bari, Distt. Jhunjhunu (R.J.), India

[2]Detptt. of Mathematics, "Sharadchandra Pawar College of Engg.", Otur (Pune), (M.S.) & Research scholar of "Singhania University" Pacheri Bari, Distt. Jhunjhunu (R.J.), India

[3]Deptt. of Mathematics, Govt. science and commerce college Benazeer, Bhopal (M.P.), India

E-mail: [1]rkbhardwaj100@gmail.com, [2]wbrlatur@gmail.com, [3]rajeshraj0101@rediffmail.com

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ABSTRACT

In the present paper some new results in topological spaces for non –symmetric rational expression conserning 2-Banach spaces are established.

Above results are motivated by Kirk, Singh and Chartarjee , Sharma and Rajputh, Yadava etal.

Keywords: *Fixed point , common fixed point , Banach Space, 2-Banach space*

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1. INTRODUCTION:

The study of Non Contraction mapping concerning the existence of fixed point draws attention of various authors in non linear analysis dealing with the study of Non –expansive mapping and the existence of fixed points. It is well known that the differential and integral equations that arise in the physical problems are generally non- linear, therefore the fixed point methods specially "Banach contraction Principle provides a powerful tool for obtaining the solution of these equations which were very difficult to solve by any other methods.

Let X be a Banach space and C a closed subset of X. The well known Banach contraction principle states that "a contraction mapping of c into itself has a unique fixed point. Ghalar (9) introduced the concept of 2 Banach space. Recently Badshah, Gupta (10) also proved the some results in 2-Banach spaces. Yadava et.al [19, 20] also worked for Banach & 2banach spaces for non –contraction mappings.

It also true that some qualitative properties of the solution of related equations are proved by functional analysis approach. Many authors have presented valuable results with non contraction mapping in Banach space. our object in this paper is to prove some fixed point and common fixed point theorem using 2-Banach space.

2 –PRELIMINARY:

Before starting main result we write some definitions.

Definition:-2.1 (2-normed space) Gahlar, has defined a linear 2- normed space as the pair $(L, \|.,.\|)$, where L is a linear space and $\|.,.\|$ is non negative real valued function defined on L such that for a, b, c ϵ L

(i)$\|a, b\| = 0$ iff a & b are linearly dependent.
(ii) $\|a, b\| = \|b, a\|$
(iii) $\|a, \beta b\| = |\beta|\|b, a\|$, β is real.
(iv) $\|a, b + c\| \leq \|a, b\| + \|a, c\|$

Hence $\|.,.\|$ is called a 2- norm. and a pair $(L, \|.,.\|)$ is called 2-Normed space.

***Corresponding author:** [1]*B .R. Wadkar*, ***E-mail:** wbrlatur@gmail.com*

Definition:-2.2 (convergent sequence) A sequence $\{x_n\}$in a linear 2- normed space L is called a convergent sequence if there is x ϵ L such that

$$\lim_{m\to\infty}\|x_n - x, y\| = 0 \text{ for all } y \,\epsilon L.$$

Definition: 2.3 (Cauchy sequence in2-normed linear space) A sequence $\{x_n\}$in a linear 2- normed space L is called a Cauchy sequence if there is y, z ϵ L such that y & z are linearly independent and the

$$\lim_{m,n\to\infty}\|x_m - x_n, y\| = 0.$$

Definition: 2.4 A linear 2-normed space in which every Cauchy sequence is convergent is called 2-Banach space.

3 MAIN RESULTS:

Theorem: 3.1 Let F be a mapping of 2- Banach space x into itself, If F satisfies the following condition:
$F^2 = I$, Where I is identity mapping

$$\begin{aligned}\|F(x)-F(y),a\| \le \alpha&\left[\frac{\|x-F(x),a\|\|y-F(y),a\|\|x-F(y),a\| + \|x-y,a\|\|y-F(x),a\|\|x-F(x),a\|}{[\|x-F(x),a\|+\|y-F(y),a\|+\|x-y,a\|]^2}\right]\\ &+\beta[\|x-F(x),a\|+\|y-F(y),a\|]\\ &+\gamma[\|x-F(y),a\|+\|y-F(x),a\|]\\ &+\delta[\|x-y,a\|]\end{aligned}$$

for every x, y $\in$ X, where $0 \le \alpha,\beta,\gamma,\delta < 1$ and $6\alpha + 4\beta + 3\gamma + \delta < 2$ then F has a fixed point, Further if $(2\gamma + \delta) < 1$ then, F has a unique fixed point.

Proof: Suppose x is point in 2- Banach space X, Taking

$$y = \frac{1}{2}(F+I)x\ ,\ z = F(y) \quad \text{and} \quad u = 2y - z$$

We obtain by hypostasis that

$$\begin{aligned}\|z-x,a\| &= \|F(y)-F^2(x),a\|\\ &= \|F(y)-FF(x),a\|\\ &\le \alpha\left[\frac{\|y-F(y),a\|\|F(x)-FF(x),a\|\|y-FF(x),a\| + \|y-F(x),a\|\|F(x)-F(y),a\|\|y-F(y),a\|}{[\|y-F(y),a\|+\|F(x)-FF(x),a\|+\|y-F(x),a\|]^2}\right]\\ &+\beta[\|y-F(y),a\|+\|F(x)-FF(x),a\|]\\ &+\gamma[\|y-FF(x),a\|+\|F(x)-F(y),a\|]\\ &+\delta[\|y-F(x),a\|]\end{aligned}$$

$$\le \alpha\left[\frac{\|y-F(y),a\|\|F(x)-x,a\|\|y-x,a\| + \|y-F(x),a\|\|F(x)-F(y),a\|\|y-F(y),a\|}{[\|y-F(y),a\|+\|F(x)-x,a\|+\|y-F(x),a\|]^2}\right]$$

$$+\beta\left[\|y-F(y),a\|+\|F(x)-x,a\|\right]$$
$$+\gamma\left[\|y-x,a\|+\|F(x)-F(y),a\|\right]$$
$$+\delta\left[\|y-F(x),a\|\right]$$

$$\leq\alpha\left[\frac{\|y-F(y),a\|\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|+\frac{1}{2}\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|\|y-F(y),a\|}{\left[\|y-F(y),a\|+\|x-y,a\|\right]^2}\right]$$
$$+\beta\left[\|y-F(y),a\|+\|F(x)-x,a\|\right]$$
$$+\gamma\left[\|y-x,a\|+\|F(x)-y+y-F(y),a\|\right]$$
$$+\delta\left[\|y-F(x),a\|\right]$$

$$\leq\alpha\left[\frac{\|y-F(y),a\|\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|+\frac{1}{2}\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|\|y-F(y),a\|}{\left[\|F(y)-x,a\|\right]^2}\right]$$
$$+\beta\left[\|y-F(y),a\|+\|F(x)-x,a\|\right]$$
$$+\gamma\left[\|y-x,a\|+\|F(x)-y,a\|+\|y-F(y),a\|\right]$$
$$+\frac{1}{2}\delta\left[\|y-F(x),a\|\right]$$

$$\leq\alpha\left[\frac{\|y-F(y),a\|\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|+\frac{1}{2}\|F(x)-x,a\|\frac{1}{2}\|F(x)-x,a\|\|y-F(y),a\|}{\left[\frac{1}{2}\|F(x)-x,a\|\right]^2}\right]$$
$$+\beta\left[\|y-F(y),a\|+\|F(x)-x,a\|\right]$$
$$+\gamma\left[\frac{1}{2}\|F(x)-x,a\|+\frac{1}{2}\|F(x)-x,a\|+\|y-F(y),a\|\right]$$
$$+\frac{1}{2}\delta\left[\|x-F(x),a\|\right]$$
$$=\alpha\left[2\|y-F(y),a\|+\|y-F(y),a\|\right]$$
$$+\beta\left[\|y-F(y),a\|+\|F(x)-x,a\|\right]$$
$$+\gamma\left[\|F(x)-x,a\|+\|y-F(y),a\|\right]$$
$$+\delta\frac{1}{2}\left[\|x-F(x),a\|\right]$$
$$=\left[3\alpha+\beta+\gamma\right]\|y-F(y),a\|+\left[\beta+\gamma+\frac{1}{2}\delta\right]\|F(x)-x,a\|$$

Therefore

$$\|z-x,a\| \le [3\alpha+\beta+\gamma]\|y-F(y),a\| + \left[\beta+\gamma+\frac{1}{2}\delta\right]\|F(x)-x,a\|$$

Also,

$$\|u-x,\alpha\| = \|2y-z-x,\alpha\|$$

$$\le \alpha\left[\frac{\|x-F(x),a\|\|y-F(y),a\|\|x-F(y),a\| + \|x-y,a\|\|y-F(x),a\|\|x-F(x),a\|}{[\|x-F(x),a\|+\|y-F(y),a\|+\|x-y,a\|]^2}\right]$$
$$+\beta[\|x-F(x),a\|+\|y-F(y),a\|]$$
$$+\gamma[\|x-F(y),a\|+\|y-F(x),a\|]$$
$$+\delta[\|x-y,a\|]$$

$$\le \alpha\left[\frac{\|x-F(x),a\|\|y-F(y),a\|\frac{1}{2}\|x-F(x),a\| + \frac{1}{2}\|x-F(x),a\|\frac{1}{2}\|x-F(x),a\|\|x-F(x),a\|}{[\|F(x)-F(y),a\|]^2}\right]$$
$$+\beta[\|x-F(x),a\|+\|y-F(y),a\|]$$
$$+\gamma\left[\frac{1}{2}\|x-F(x),a\|+\frac{1}{2}\|x-F(x),a\|\right]$$
$$+\frac{1}{2}\delta[\|x-F(x),a\|]$$

$$\le \alpha\left[\frac{\|x-F(x),a\|\|y-F(y),a\|\frac{1}{2}\|x-F(x),a\| + \frac{1}{2}\|x-F(x),a\|\frac{1}{2}\|x-F(x),a\|\|x-F(x),a\|}{\left[\frac{1}{2}\|x-F(x)\|,a\right]^2}\right]$$
$$+\beta[\|x-F(x),a\|+\|y-F(y),a\|]$$
$$+\gamma[\|x-F(x),a\|]+\frac{1}{2}\delta[\|x-F(x),a\|]$$
$$=[2\alpha+\beta]\|y-F(y),a\| + \left[\alpha+\beta+\gamma+\frac{1}{2}\delta\right]\|x-F(x),a\|$$

Now

$$\|z-u,a\| \le \|z-x,a\|+\|x-u,a\|$$
$$=[3\alpha+\beta+\gamma]\|y-F(y),a\| + \left[\beta+\gamma+\frac{1}{2}\delta\right]\|F(x)-x,a\|$$
$$+[2\alpha+\beta]\|y-F(y),a\| + \left[\alpha+\beta+\gamma+\frac{1}{2}\delta\right]\|x-F(x),a\|$$
$$=[5\alpha+2\beta+\gamma]\|y-F(y),a\| + [\alpha+2\beta+2\gamma+\delta]\|x-F(x),a\|$$

Also,

$$\|z-u,a\| = \|F(y)-2y+z,a\| = 2\|F(y)-y,a\|$$

$$2\|F(y)-y,a\| \le [5\alpha+2\beta+\gamma]\|y-F(y),a\| + [\alpha+2\beta+2\gamma+\delta]\|x-F(x),a\|$$

$$\Rightarrow \|F(y)-y,a\| \le \left[\frac{5}{2}\alpha+\beta+\frac{1}{2}\gamma\right]\|y-F(y),a\| + \left[\frac{1}{2}\alpha+\beta+2\gamma+\frac{1}{2}\delta\right]\|x-F(x),a\|$$

$$\Rightarrow \|y-f(y),a\| \le \left[\frac{\frac{\alpha}{2}+\beta+\gamma+\frac{\delta}{2}}{1-\frac{5}{2}\alpha-\beta-\frac{\gamma}{2}}\right]\|x-F(x),a\|$$

$$\Rightarrow \|y-f(y),a\| \le q\|x-F(x),a\|$$

$$\text{where } q = \frac{\frac{\alpha}{2}+\beta+\gamma+\frac{\delta}{2}}{1-\frac{5}{2}\alpha-\beta-\frac{\gamma}{2}} < 1$$

Since

$$6\alpha+4\beta+3\gamma+\delta<2$$

On taking

$$G = \frac{1}{2}(F+I) \text{ for every x } \varepsilon \text{ X}$$

$$\left[\|G^2(x)-G(x),a\| = \|G(y)-y,a\| = \left\|\frac{1}{2}(F+I)y-y,a\right\|\right] = \frac{1}{2}\|y-F(y),a\| \le q\|x-F(x),a\|$$

By definition of q we claim that $\{G^n(x)\}$ is a cauchy sequence in X. Therefore by the property of compactnes s.$\{G^n(x)\}$ converges to some element x_0 in X

$$\lim_{n\to\infty} G^n(x) = x_0$$

$$\Rightarrow G(x_0) = x_0$$

$$\Rightarrow F(x_0) = x_0$$

i.e. x_0 is a fixed point of F

For uniqueness , if possible let $y_0 (\ne x_0)$ be another fixed point of F then

$$\|x_0-y_0,a\| = \|F(x_0)-F(y_0),a\|$$

$$\le \alpha\left[\frac{\|x_0-F(x_0),a\|\|y_0-F(y_0),a\|\|x_0-F(y_0),a\| + \|x_0-y_0,a\|\|y_0-F(x_0),a,\|\|xo-F(x0),a\|}{[\|x_0-F(x_0),a\|+\|y_0-F(y_0),a\|+\|x_0-y_0,a\|]^2}\right]$$

$$+\beta[\|x_0-F(x_0),a\|+\|y_0-F(y_0),a\|]$$
$$+\gamma[\|x_0-F(y_0),a\|+\|y_0-F(x_0),a\|]+\delta[\|x_0-y_{0,a}\|]$$

$$= \beta\left[\left\|y_0 - F(y_0), a\right\|\right]$$
$$+ \gamma\left[\left\|x_0 - F(y_0), a\right\| + \left\|y_0 - F(x_0), a\right\|\right] + \delta\left[\left\|x_0 - y_{0,a}\right\|\right]$$
$$= \gamma\left[\left\|x_0 - F(y_0), a\right\| + \left\|y_0 - F(x_0), a\right\|\right] + \delta\left[\left\|x_0 - y_{0,a}\right\|\right]$$
$$= \gamma\left[\left\|x_0 - y_{0,a}\right\| + \left\|y_0 - x_{0,a}\right\|\right] + \delta\left[\left\|x_0 - y_{0,a}\right\|\right]$$
$$= (2\gamma + \delta)\left[\left\|x_0 - y_{0,a}\right\|\right]$$

since $(2\gamma+\delta) < 1$, there fore $\left\|x_0 - y_0\right\| = 0$

$\Rightarrow x_0 = y_0$

This completes the proof of theorem: 3.1

Now we prove the following theorem which generalize the theorem: 3.1

Theorem: 3.2
let K be closed and convex subset of a 2 - Banach space X. Let F : K $\rightarrow$ K, G : K $\rightarrow$ K satisfy the conditions

F and G commutes (3.2.1)

F^2 = I and G^2 = I , Where I isdenotes the identity mapping (3.2.2)

$$\left\|F(x) - F(y), a\right\| \le \alpha \left[\frac{\left\|G(x) - F(x), a\right\|\left\|G(y) - F(y), a\right\|\left\|G(x) - F(y), a\right\| + \left\|G(x) -, G(y), a\right\|\left\|G(y) - F(x), a\right\|\left\|G(x) - F(x), a\right\|}{\left[\left\|G(x) - F(x), a\right\| + \left\|G(y) - F(y), a\right\| + \left\|G(x) - G(y), a\right\|\right]^2} \right]$$
$$+ \beta\left[\left\|G(x) - F(x), a\right\| + \left\|G(y) - F(y), a\right\|\right]$$
$$+ \gamma\left[\left\|G(x) - F(y), a\right\| + \left\|G(y) - F(x), a\right\|\right]$$
$$+ \delta\left[\left\|G(x) - G(y), a\right\|\right] \tag{3.2.3}$$

For every x, y $\in$ X, $0 \le \alpha, \beta, \delta, \gamma$ with $6\alpha + 4\beta + 3\gamma + \delta < 2$ then there exit atleast one fixed point $x_0 \varepsilon$ X such that $F(x_0) = G(x_0) = x_0$. Further if $(2\gamma + \delta) < 1$ then x_0 is the unique fixed point of F and G.

Proof: From (3.2.1) and (3.2.2) it follows that $(FG)^2 = I$ and (3.2.2) and (3.2.3) implies

$$\left\|FGG(x) - FGG(y), a\right\| = \left\|FG^2(x) - FG^2(y), a\right\|$$
$$= \alpha \left[\frac{\left\|GG^2(x) - FG^2(x), a\right\|\left\|GG^2(y) - FG^2(y), a\right\|\left\|GG^2(x) - FG^2(y), a\right\| + \left\|GG^2(x) - GG^2(y), a\right\|\left\|GG^2(y) - FG^2(x), a\right\|\left\|GG^2(x) - FG^2(x), a\right\|}{\left[\left\|GG^2(x) - FG^2(x), a\right\| + \left\|GG^2(y) - FG^2(y), a\right\| + \left\|GG^2(x) - GG^2(y), a\right\|\right]^2} \right]$$
$$+ \beta\left[\left\|GG^2(x) - FG^2(x), a\right\| + \left\|GG^2(y) - FG^2(y), a\right\|\right]$$
$$+ \gamma\left[\left\|GG^2(x) - FG^2(y), a\right\| + \left\|GG^2(y) - FG^2(x), a\right\|\right]$$
$$+ \delta\left\|GG^2(x) - GG^2(y), a\right\|$$

$$= \alpha \left[\frac{\|G(x) - FG.G(x), a\| \|G(y) - FG.G(y), a\| \|G(x) - FG.G(y), a\| + \|G(x) - G(y), a\| \|G(y) - FG.G(x), a\| \|G(x) - FG.G(x), a\|}{\left[\|G(x) - FG.G(x), a\| + \|G(y) - FG.G(y), a\| + \|G(x) - G(y), a\|\right]^2} \right]$$
$$+ \beta\left[\|G(x) - FG.G(x), a\| + \|G(y) - FG.G(y), a\|\right]$$
$$+ \gamma\left[\|G(x) - FG.G(y), a\| + \|G(y) - FG.G(x), a\|\right]$$
$$+ \delta\|G(x) - G(y), a\|$$

Now we put

G(x) = z and G(y) = w then we get

$$\|FG(z) - FG(w), a\| \leq \left[\frac{\|z - FG(z), a\| \|w - FG(w), a\| \|z - FG(w), a\| + \|z - w, a\| \|w - FG(z), a\| \|z - FG(z), a\|}{\left[\|z - FG(z), a\| + \|w - FG(w), a\| + \|z - w, a\|\right]^2} \right]$$
$$+ \beta\left[\|z - FG(z), a\| + \|w - FG(w), a\|\right]$$
$$+ \gamma\left[\|z - FG(w), a\| + \|w - FG(z), a\|\right]$$
$$+ \delta\|z - w, a\|$$

we have $(FG)^2 = I$ and so by theorem-3.1, FG has at least one fixed point say x_0 in K

$$i.e\ F(x_0) = x_0 \tag{3.2.4}$$

$$\text{and so } F.(FG)x_0 = F(x_0)$$

$$G(x_0) = F(x_0) \tag{3.2.5}$$

Now,

$$\|F(x_0) - x_0, a\| = \|F(x_0) - F^2(x_0), a\|$$
$$= \|F(x_0) - FF(x_0), a\|$$
$$\leq \alpha \left[\frac{\|G(x_0) - F(x_0), a\| \|GF(x_0) - FF(x_0), a\| \|G(x_0) - FF(x_0), a\| + \|G(x_0) - GF(x_0), a\| \|GF(x_0) - F(x_0), a\| \|G(x_0) - F(x_0), a\|}{\left[\|G(x_0) - F(x_0), a\| + \|GF(x_0) - FF(x_0), a\| + \|G(x_0) - GF(x_0), a\|\right]^2} \right]$$
$$+ \beta\left[\|G(x_0) - F(x_0), a\| + \|GF(x_0) - FF(x_0), a\|\right]$$
$$+ \gamma\left[\|G(x_0) - FF(x_0), a\| + \|GF(x_0) - F(x_0), a\|\right]$$
$$+ \delta\|G(x_0) - GF(x_0), a\|$$

$$= \alpha \left[\frac{\begin{array}{c} \|G(x_0) - F(x_0), a\| \|x_0 - x_0, a\| \|G(x_0) - x_0, a\| \\ + \|G(x_0) - x_0, a\| \|x_0 - F(x_0), a\| \|G(x_0) - F(x_0), a\| \end{array}}{[\|G(x_0) - F(x_0), a\| + \|x_0 - x_0, a\| + \|G(x_0) - x_0, a\|]^2} \right]$$

$$+ \beta \left[\|G(x_0) - F(x_0), a\| + \|x_0 - x_0, a\| \right]$$

$$+ \gamma \left[\|G(x_0) - x_0, a\| + \|x_0 - F(x_0), a\| \right]$$

$$+ \delta \|G(x_0) - x_0, a\|$$

$$= \alpha \left[\frac{\begin{array}{c} \|F(x_0) - F(x_0), a\| \|x_0 - x_0, a\| \|F(x_0) - x_0, a\| \\ + \|F(x_0) - x_0, a\| \|x_0 - F(x_0), a\| \|F(x_0) - F(x_0), a\| \end{array}}{[\|F(x_0) - F(x_0), a\| + \|x_0 - x_0, a\| + \|F(x_0) - x_0, a\|]^2} \right]$$

$$+ \beta \left[\|F(x_0) - F(x_0), a\| + \|x_0 - x_0, a\| \right]$$

$$+ \gamma \left[\|F(x_0) - x_0, a\| + \|x_0 - F(x_0), a\| \right]$$

$$+ \delta \|F(x_0) - x_0, a\|$$

$$= (2\gamma + \delta) \|F(x_0) - x_0, a\|$$

since $(2\gamma + \delta) < 1$ it follows that $F(x_0) = x_0$

i,e. x_0 is fixed point of F, but $F(x_0) = G(x_0)$ there fore we have $G(x_0) = x_0$

i.e. x_0 is common fixed point of F and G

Now we shall prove that x_0 is unique common fixed point of F and G. If possible , let y_0 be another fixed point of F and G

Now from (3.2.1), (3.2.2), (3.2.3), (3.2.4)and (3.2.5) we have

Now from (3.2.1), (3.2.2), (3.2.3), (3.2.4)and (3.2.5) we have

$$\|x_0 - y_0, a\| = \|F^2(x_0) - F^2(y_0), a\|$$

$$= \|FF(x_0) - FF(y_0), a\|$$

$$\leq \alpha \left[\frac{\begin{array}{c} \|GF(x_0) - FF(x_0), a\| \|GF(y_0) - FF(y_0), a\| \|GF(x_0) - FF(y_0), a\| \\ + \|GF(x_0) - GF(y_0), a\| \|GF(y_0) - FF(x_0), a\| \|GF(x_0) - FF(x_0), a\| \end{array}}{[\|GF(x_0) - FF(x_0), a\| + \|GF(y_0) - FF(y_0), a\| + \|GF(x_0) - FF(y_0), a\|]^2} \right]$$

$$+ \beta \left[\|GF(x_0) - FF(x_0), a\| + \|GF(y_0) - FF(y_0), a\| \right]$$

$$+ \gamma \left[\|GF(x_0) - FF(y_0), a\| + \|GF(y_0) - FF(x_0), a\| \right]$$

$$+ \delta \|GF(x_0) - GF(y_0), a\|$$

$$\leq \alpha\left[\frac{\|x_0-x_0,a\|\|y_0-y_0,a\|\|x_0-y_0,a\|+\|x_0-y_0,a\|\|y_0-x_0,a\|\|x_0-x_0,a\|}{[\|x_0-x_0,a\|+\|y_0-y_0,a\|+\|x_0-y_0,a\|]^2}\right]$$
$$+\beta[\|x_0-x_0,a\|+\|y_0-y_0,a\|]$$
$$+\gamma[\|x_0-y_0,a\|+\|y_0-x_0,a\|]$$
$$+\delta[\|x_0-y_0,a\|]$$
$$=(2\gamma+\delta)\|x_0-y_0,a\|$$

Therefore

$$\|x_0-y_0,a\|\leq(2\gamma+\delta)\|x_0-y_0,a\|$$

Since $(2\gamma+\delta)<1$, it follows that $x_0=y_0$, proving the uniqueness of x_0.

The proof of the theorem 3.2 is comleted.

Nowwe shall generalize theorem 3.2 by takin three non –contraction typemapping F, G &H in 2- Banach space.

Theorem - 3.3
Let F , G and H be three mappings of 2-Banach space X into itself such that

$$\mathrm{FG}=\mathrm{GF}, \mathrm{GH}=\mathrm{HG} \text{ and } \mathrm{FH}-\mathrm{HF} \tag{3.3.1}$$
$$F^2=I, G^2=I, H^2=I$$

Where I is identity mapping

$$\|F(x)-F(y)\|\leq\alpha\left[\frac{\|GH(x)-F(x),a\|\|GH(y)-F(y),a\|\|GH(x)-F(y),a\| + \|GH(x)-GH(y),a\|\|GH(y)-F(x),a\|\|GH(x)-F(x),a\|}{[\|GH(x)-F(x),a\|+\|GH(y)-F(y),a\|+\|GH(x)-GH(y),a\|]^2}\right] \tag{3.3.2}$$
$$=\beta[\|GH(x)-F(x),a\|+\lfloor GH(y)-F(y),a\rfloor]$$
$$+\gamma[\|GH(x)-F(y),a\|+\lfloor GH(y)-F(x),a\rfloor]$$
$$+\delta[\|GH(x)-GH(y),a\|] \tag{3.3.3}$$

for every x, $y\in X$ and $0\leq\alpha,\beta,\gamma,\delta,<1$ such that $6\alpha+4\beta+3\gamma+\delta<2$ then F, G and H have atleast one fixed point.Furt her if $(2\gamma+\delta)<1$, then there exist unique fixed point of F, G and H.

Proof:

From (3.3.1) and (3.3.2) it follows that $(\mathrm{FGH})^2=I$, where I is identity mapping.
From (3.3.2) and (3.3.3) ,we have

$$\|FGH.G(x)-FGH.G(y),a\| \le$$

$$\alpha\left[\frac{\|(GH)^2G(x)-FGHG(x),a\|\|(GH)^2G(y)-FGHG(y),a\|\|(GH)^2G(x)-FGHG(y),a\| + \|(GH)^2G(x)-(GH)^2G(y),a\|\|(GH)^2G(y)-FGHG(x),a\|\|(GH)^2G(x)-FGHG(x),a\|}{\left[\|(GH)^2G(x)-FGHG(x),a\|+\|(GH)^2G(y)-FGHG(y),a\|+\|(GH)^2G(x)-(GH)^2G(y),a\|\right]^2}\right]$$

$$+\beta\left[\|(GH)^2G(x)-FGHG(x),a\|+\|(GH)^2G(y)-FGHG(y),a\|\right]$$
$$+\gamma\left[\|(GH)^2G(x)-FGHG(y),a\|+\|(GH)^2G(y)-FGHG(x),a\|\right]$$
$$+\delta\left[\|(GH)^2G(x)-(GH)^2G(y),a\|\right]$$

$$\|FGH.G(x)-FGH.G(y),a\| \le \alpha\left[\frac{\|G(x)-FGHG(x),a\|\|G(y)-FGHG(y),a\|\|G(x)-FGHG(y),a\| + \|G(x)-G(y),a\|\|G(y)-FGHG(x),a\|\|G(x)-FGHG(x),a\|}{\left[\|G(x)-FGHG(x),a\|+\|G(y)-FGHG(y),a\|+\|G(x)-G(y),a\|\right]^2}\right]$$

$$+\beta\left[\|G(x)-FGHG(x),a\|+\|G(y)-FGHG(y),a\|\right]$$
$$+\gamma\left[\|G(x)-FGHG(y),a\|+\|G(y)-FGHG(x),a\|\right]$$
$$+\delta\left[\|G(x)-G(y),a\|\right]$$

If we put G(x) = z and G(y) = w, we get

$$\|FGH.(z)-FGH.(w),a\| \le \alpha\left[\frac{\|z-FGH(z),a\|\|w-FGH(w),a\|\|z-FGH(w),a\| + \|z-w,a\|\|w-FGH(z),a\|\|z-FGH(z),a\|}{\left[\|z-FGH(z)),a\|+\|w-FGH(w),a\|+\|z-w,a\|\right]^2}\right]$$

$$+\beta\left[\|z-FGH(z)),a\|+\|w-FGH(w),a\|\right]$$
$$+\gamma\left[\|z-FGH(w),a\|+\|w-FGH(z),a\|\right]$$
$$+\delta\left[\|z-w,a\|\right]$$

If we FGH = N, we get

$$\|N(z)-N(w),a\| \le \alpha\left[\frac{\|z-N(z),a\|\|w-N(w),a\|\|z-N(w),a\| + \|z-w,a\|\|w-N(z),a\|\|z-N(z),a\|}{\left[\|z-N(z),a\|+\|w-N(w),a\|+\|z-w,a\|\right]^2}\right]$$

$$+\beta\left[\|z-N(z),a\|+\|w-N(w),a\|\right]$$
$$+\gamma\left[\|z-N(w),a\|+\|w-N(z),a\|\right]$$
$$+\delta\left[\|z-w,a\|\right]$$

Let $w = \frac{1}{2}(N+I)z$, & $N(w) = s$ & $t = 2w - s$

(3.3.4)

Now by (3.3.4), we have

$$\|s - z, a\| = \|N(w) - z, a\|$$
$$= \|N(w) - N^2(z), a\|$$
$$= \|N(w) - NN(z), a\|$$
$$\leq \alpha\left[\frac{\|w - N(w), a\|\|N(z) - NN(z), a\|\|w - NN(z), a\| + \|w - N(z), a,\|\|N(z) - N(w), a\|\|w - N(w), a\|}{[\|w - N(w), a\| + \|N(z) - NN(z), a\| + \|w - N(z), a\|]^2}\right]$$
$$+ \beta[\|w - N(w), a\| + \|N(z) - NN(z), a\|]$$
$$+ \gamma[\|w - NN(z), a\| + \|N(z) - N(w), a\|]$$
$$+ \delta[\|w - N(z), a\|]$$
$$\leq \alpha\left[\frac{\|w - N(w), a\|\|N(z) - z, a\|\|w - z), a\| + \|w - N(z), a\|\|N(z) - N(w), a\|\|w - N(w), a\|}{[\|w - N(w), a\| + \|N(z) - z, a\| + \|w - N(z), a\|]^2}\right]$$
$$+ \beta[\|w - N(w), a\| + \|N(z) - z, a\|]$$
$$+ \gamma[\|w - z, a\| + \|N(z) - w, a\| + +\|w - N(w), a\|]$$
$$+ \delta[\|w - N(z), a\|]$$
$$\leq \alpha\left[\frac{\|w - N(w), a\|\|N(z) - z, a\|\left\|\frac{1}{2}(N+I)z - z), a\right\| + \left\|\frac{1}{2}(N+I)z - N(z), a\right\|\left\|N(z) - N(\frac{1}{2}(N+I)z), a\right\|\|w - N(w), a\|}{[\|w - N(w), a\| + \|w - z, a\|]^2}\right]$$
$$+ \beta[\|w - N(w), a\| + \|N(z) - z, a\|]$$
$$+ \gamma\left[\left\|\frac{1}{2}(N+I)z - z, a\right\| + \left\|N(z) - \frac{1}{2}(N+I)z, a\right\| + \|w - N(w), a\|\right]$$
$$+ \delta\left[\left\|\frac{1}{2}(N+I)z - N(z), a\right\|\right]$$
$$\leq \alpha\left[\frac{\|w - N(w), a\|\|N(z) - z, a\|\frac{1}{2}\|N(z) - z), a\| + \frac{1}{2}\|N(z) - z), a\|\frac{1}{2}\|N(z) - z), a\|\|w - N(w), a\|}{[\|N(w) - z, a\|]^2}\right]$$
$$+ \beta[\|w - N(w), a\| + \|N(z) - z, a\|]$$
$$+ \gamma\left[\frac{1}{2}\|N(z) - z), a\| + \frac{1}{2}\|N(z) - z), a\| + \|w - N(w), a\|\right]$$
$$+ \delta\left[\frac{1}{2}\|N(z) - z), a\|\right]$$

$$\leq \alpha\left[\frac{\|w-N(w),a\|\|N(z)-z,a\|\frac{1}{2}\|N(z)-z),a\| + \frac{1}{2}\|N(z)-z),a\|\frac{1}{2}\|N(z)-z),a\|\|w-N(w),a\|}{\left[\left\|N(\frac{1}{2}(N+I)z-z,a\right\|\right]^2}\right]$$
$$+\beta\left[\|w-N(w),a\|+\|N(z)-z,a\|\right]$$
$$+\gamma\left[\|N(z)-z),a\|+\|w-N(w),a\|\right]$$
$$+\delta\left[\frac{1}{2}\|N(z)-z),a\|\right]$$

$$\leq \alpha\left[\frac{\|w-N(w),a\|\|N(z)-z,a\|\frac{1}{2}\|N(z)-z),a\| + \frac{1}{2}\|N(z)-z),a\|\frac{1}{2}\|N(z)-z),a\|\|w-N(w),a\|}{\left[\frac{1}{2}\|N(z)-z),a\|\right]^2}\right]$$
$$+\beta\left[\|w-N(w),a\|+\|N(z)-z,a\|\right]$$
$$+\gamma\left[\|N(z)-z),a\|+\|w-N(w),a\|\right]$$
$$+\delta\left[\frac{1}{2}\|N(z)-z),a\|\right]$$

$$\leq \alpha\left[2\|w-N(w),a,\|+\|w-N(w),a\|\right]$$
$$+\beta\left[\|w-N(w),a\|+\|N(z)-z,a\|\right]$$
$$+\gamma\left[\|N(z)-z,a\|+\|w-N(w),a\|\right]$$
$$+\delta\frac{1}{2}\|N(z)-z,a\|$$
$$\leq (3\alpha+\beta+\gamma)\|w-N(w),a\|+(\beta+\gamma+\frac{\delta}{2})\|N(z)-z,a\|$$

Therefore

$$\|s-z,a\| \leq (3\alpha+\beta+\gamma)\|w-N(w),a\|+(\beta+\gamma+\frac{\delta}{2})\|N(z)-z,a\| \tag{3.3.5}$$

Now,

$$\|\mathrm{t-z,a}\| \leq \|2w-s-z,a\|$$
$$=\left\|2(\frac{1}{2}(N+I)z-s-z,a\right\|$$
$$=\|N(z)+z-s-z,a\|$$
$$=\|N(z)-s,a\|$$
$$=\|N(z)-N(w),a\|$$

$$\leq \alpha\left[\frac{\|z-N(z),a\|\|w-N(w),a\|\|z-N(w),a\|+\|z-w,a\|\|w-N(z),a\|\|z-N(z),a\|}{[\|z-N(z),a\|+\|w-N(w),a\|+\|z-w,a\|]^2}\right]$$
$$+\beta[\|z-N(z),a\|+\|w-N(w),a\|]$$
$$+\gamma[\|z-N(w),a\|+\|w-N(z),a\|]$$
$$+\delta\|z-w,a\|$$

$$\leq \alpha\left[\frac{\begin{array}{l}\|z-N(z),a\|\|w-N(w),a\|\left\|z-N(\frac{1}{2}(N+I)z),a\right\| \\ +\left\|z-\frac{1}{2}(N+I)z,a\right\|\left\|\frac{1}{2}(N+I)z-N(z),a\right\|\|z-N(z),a\|\end{array}}{[\|N(z)-N(w),a\|]^2}\right]$$
$$+\beta[\|z-N(z),a\|+\|w-N(w),a\|]$$
$$+\gamma\left[\left\|z-N(\frac{1}{2}(N+I)z,a\right\|+\left\|\frac{1}{2}(N+I)z-N(z),a\right\|\right]$$
$$+\delta\left\|z-\frac{1}{2}(N+I)z,a\right\|$$

$$\leq \alpha\left[\frac{\|z-N(z),a\|\|w-N(w),a\|\frac{1}{2}\|z-N(z),a\|+\frac{1}{2}\|z-N(z),a\|\frac{1}{2}\|z-N(z),a\|\|z-N(z),a\|}{\left[\frac{1}{2}\|z-N(z),a\|\right]^2}\right]$$
$$+\beta[\|z-N(z),a\|+\|w-N(w),\|]$$
$$+\gamma\left[\frac{1}{2}\|z-N(z),a\|+\frac{1}{2}\|z-N(z),a\|\right]$$
$$+\delta\frac{1}{2}\|z-N(z),a\|$$
$$=\alpha[2\|w-N(w),a\|+\|z-N(z),a\|]$$
$$+\beta[\|w-N(w),a\|+\|z-N(z),a\|]$$
$$+\gamma\|z-N(z),a\|$$
$$+\delta\frac{1}{2}\|z-N(z),a\|$$

$$=(2\alpha+\beta)\|w-N(w),a\|+(\alpha+\beta+\gamma+\frac{1}{2}\delta)\|z-N(z),a\| \tag{3.3.6}$$

Now

$$\|s-t,a\|\leq\|s-z,a\|+\|z-t,a\|$$

Therefore from (3.3.5) &(3.3.6), we have,

$$\|s-t,a\| \le (3\alpha+\beta+\gamma)\|w-N(w),a\| + (\beta+\gamma+\frac{\delta}{2})\|N(z)-z,a\|$$
$$+(2\alpha+\beta)\|w-N(w),a\| + (\alpha+\beta+\gamma+\frac{1}{2}\delta)\|z-N(z),a\|$$
$$\le (5\alpha+2\beta+\gamma)\|w-N(w),a\| + (\alpha+2\beta+2\gamma+\delta)\|z-N(z),a\| \tag{3.3.7}$$

$$\|s-t,a\| = \|N(w)-(2w-s),a\|$$
$$= \|N(w)-2w+N(w),a\|$$
$$= 2\|N(w)-w,a\|$$

Putting all these in inequality (3.3.7), we get,

$$2\|N(w)-w,a\| \le 5\alpha+2\beta+\gamma)\|w-N(w),a\| + (\alpha+2\beta+2\gamma+\delta)\|z-N(z),a\|$$
$$\Rightarrow \|N(w)-w,a\| \le (\frac{5}{2}\alpha+\beta+\frac{\gamma}{2})\|w-N(w),a\| + (\frac{1}{2}\alpha+\beta+\gamma+\frac{1}{2}\delta)\|z-N(z),a\|$$
$$\Rightarrow \|N(w)-w,a\| \le q\|z-N(z),a\|$$

$$\text{where } q = \frac{\frac{1}{2}\alpha+\beta+\gamma+\frac{1}{2}\delta}{1-\frac{5}{2}\alpha-\beta-\frac{\gamma}{2}} < 1$$

since $5\alpha+4\beta+3\gamma+\delta<2$

on taking $G = \frac{1}{2}(N+I)$ then for any $z \varepsilon X$

$$\|G^2(z)-G(z),a\| = \|G(w)-w,a\|$$
$$= \left\|\frac{1}{2}(N+I)w-w,a\right\|$$
$$= \frac{1}{2}\|N(w)-w,a\|$$
$$\le \frac{q}{2}\|z-N(z),a\|$$

By definition of q we claim that $\{G^n(x)\}$ is a Cauchy sequence in X. By compactness, $\{G^n(x)\}$ converges to some element x_0 in X.

i.e. $\lim_{n\to\infty} G^n(x_0) = x_0$

$\Rightarrow G(x_0) = x_0$

Hence $N(x_0) = x_0$

Or $FGH(x_0) = x_0$ because N=FGH (3.3.8)

And so $GH(FGH)(x_0) = GH(x_0)$

or $F(x_0) = GH(x_0)$ (3.3.9)

Also $H(FGH)(x_0) = (x_0)$

$FG(x_0) = H(x_0)$ (3.3.10)

Now using (1),(2),(3),and (3.3.8),(3.3.9) &(3.3.10), we have

$$\|H(x_0)-x_0,a\| = \|FG(x_0)-F^2(x_0),a\|$$
$$= \|FG(x_0)-F(F(x_0)),a\|$$
$$\leq \alpha\left[\frac{\|GHG(x_0)-FG(x_0),a\|\|GHF(x_0)-FF(x_0),a\|\|GHG(x_0)-FG(x_0),a\| + \|GHG(x_0)-GHF(x_0),a\|\|GHF(x_0)-FG(x_0),a\|\|GHG(x_0)-FG(x_0),a\|}{[\|GHG(x_0)-FG(x_0),a\|+\|GHF(x_0)-FF(x_0),a\|+\|GHG(x_0)-GHF(x_0),a\|]^2}\right]$$
$$+\beta[\|GHG(x_0)-FG(x_0),a\|+\|GHF(x_0)-FF(x_0),a\|]$$
$$+\gamma[\|GHG(x_0)-FF(x_0),a\|+\|GHF(x_0)-FG(x_0),a\|]$$
$$+\delta\|[\|GHG(x_0)-GHF(x_0),a\|]\|$$
$$= (2\gamma+\delta)\|H(x_0)-x_0,a\|$$

Hence $\|H(x_0)-x_0,a\| \leq (2\gamma+\delta)\|H(x_0)-x_0,a\|$

since $(2\gamma+\delta)<1$, it follows that $H(x_0)=x_0$ i.e x_0 is the fixed point of H .

Thus we have from (3.3.9) , G(x_0) = F(x_0)

Again

$$\|F(x_0)-x_0,a\| = \|F(x_0)-F^2(x_0),a\|$$
$$= \|F(x_0)-FF(x_0),a\|$$
$$\leq \alpha\left[\frac{\|GH(x_0)-F(x_0),a\|\|GHF(x_0)-FF(x_0),a\|\|GH(x_0)-FF(x_0),a\| + \|GH(x_0)-GHF(x_0),a\|\|GHF(x_0)-F(x_0),a\|\|GH(x_0)-F(x_0),a\|}{[\|GH(x_0)-F(x_0),a\|+\|GH(x_0)-FF(x_0),a\|+\|GH(x_0)-GHF(x_0),a\|]^2}\right]$$
$$+\beta[\|GH(x_0)-F(x_0),a\|+\|GHF(x_0)-FF(x_0),a\|]$$
$$+\gamma[\|GH(x_0)-FF(x_0),a\|+\|GHF(x_0)-F(x_0),a\|]$$
$$+\delta[\|GH(x_0)-GHF(x_0),a\|]$$

$$\alpha\left[\frac{\|F(x_0)-F(x_0),a\|\|x_0-x_0,a\|\|F(x_0)-x_0,a\| + \|F(x_0)-x_0,a\|\|x_0-F(x_0),a\|\|F(x_0)-F(x_0),a\|}{[\|F(x_0)-F(x_0),a\|+\|x_0-x_0,a\|+\|F(x_0)-x_0,a\|]^2}\right]$$
$$+\beta[\|F(x_0)-F(x_0),a\|+\|x_0-x_0,a\|]$$
$$+\gamma[\|F(x_0)-x_0,a\|+\|x_0-F(x_0),a\|]$$
$$+\delta[\|F(x_0)-x_0,a\|]$$
$$= (2\gamma+\delta)\|F(x_0)-x_0,a\|$$

$$\|F(x_0)-x_0,a\| \leq (2\gamma+\delta)\|F(x_0)-x_0,a\|$$

which is contradiction ,since $(2\gamma+\delta)<1$. Hence it follows that :

$F(x_0) = x_0$

But $F(x_0) = G(x_0)$

Therefore $F(x_0) = G(x_0) = x_0$

i.e. x_0 is the common fixed point of F ,G and H . Now to conform the uniqueness of x_0 let y_0 be another common fixed point of F ,G & H using (1),(2),(3) and (A),(B),(C) we get,

$$\begin{aligned}\|x_0 - y_0, a\| &= \|F^2(x_0) - F^2(y_0), a\|\\ &= \|FF(x_0) - FF(y_0), a\|\\ &= \|F(F(x_0)) - F(F(y_0)), a\|\end{aligned}$$

$$\leq \alpha\left[\frac{\begin{array}{c}\|GHF(x_0)-FF(x_0),a\|\|GHF(y_0)-FF(y_0),a\|\|GHF(x_0)-FF(y_0),a\|\\ +\|GHF(x_0)-GHF(y_0),a\|\|GHF(y_0)-FF(x_0),a\|\|GHF(x_0)-FF(x_0),a\|\end{array}}{\left[\|GHF(x_0)-FF(x_0),a\|+\|GHF(y_0)-FF(y_0),a\|+\|GHF(x_0)-GHF(y_0),a\|\right]^2}\right]$$
$$+\beta\left[\|GHF(x_0)-FF(x_0),a\|+\|GHF(y_0)-FF(y_0),a\|\right]$$
$$+\gamma\left[\|GHF(x_0)-FF(y_0),a\|+\|GHF(y_0)-FF(x_0),a\|\right]$$
$$+\delta\left[\|GHF(x_0)-GHF(y_0),a\|\right]$$
$$\leq \alpha\left[\frac{\begin{array}{c}\|x_0-x_0,a\|\|y_0-y_0,a\|\|x_0-y_0,a\|\\ +\|x_0-y_0,a\|\|y_0-x_0,a\|\|x_0-x_0,a\|\end{array}}{\left[\|x_0-x_0,a\|+\|y_0-y_0,a\|+\|x_0-y_0,a\|\right]^2}\right]$$
$$+\beta\left[\|x_0-x_0,a\|+\|y_0-y_0,a\|\right]$$
$$+\gamma\left[\|x_0-y_0,a\|+\|y_0-x_0,a\|\right]$$
$$+\delta\left[\|x_0-y_0,a\|\right]$$
$$=(2\gamma+\delta)\left[\|x_0-y_0,a\|\right]$$

Therefore

$$\|x_0-y_0,a\| \leq (2\gamma+\delta)\|x_0-y_0,a\|$$

Which is contradiction, since $(2\gamma+\delta)<1$

Hence it follow that $x_0 = y_0$, proving the uniqueness of x_0 .

This completes the proof of the theorem 3.3

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