# International Research Journal of Pure Algebra -4(4), 2014, 471-476 CRPA <br> Available online through www.rjpa.info ISSN 2248-9037 

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(Received on: 26-03-14; Revised \& Accepted on: 05-04-14)


#### Abstract

In this paper we come across hexagonal sgrafitto automata. The induced properties on picture languages are considered herewith. It also contains discussion on time complexity.


Subject Classification: Applied Mathematics.
Key Words: Hexagonal Sgrafitto Automaton, Time Complexity, Determinism, Finite state Automaton.

## 1. INTRODUCTION

There has been works related to picture languages in the last three decades. Many different mechanisms were considered. In [3] the notion of local picture languages using tiling system were discussed and projection of local picture languages. In [6] the hexagonal tiling system was introduced and defines local as well as the recognizable hexagonal picture languages.

Among several extensions to the two dimensions case of the well-known notion of recognizability it can be defined by other equivalent ways as for example by online tessellation automata [5]. In [11] hexagonal tessellation automata and 6NFA and 6DFA and their relation with hexagonal picture languages are discussed. A Wang hexagonal automaton combines features of both online tessellation acceptors computation assigns states to each hexagonal picture positions in 6 way automata. The input head visits the hexagonal picture moving from one pixel to an adjacent one according to some scanning strategy.

The choice of scanning strategy is the central issue. In [12] we introduce a polite scanning strategy that sort all positions on the picture and visits each of them exactly once in such a way that the next position to visit is always adjacent to the previous one and opens only on this information with neighboring positions already visited.

In this paper a new model called Hexagonal Sgrafitto Automata is introduced. The studies have been done on induced properties on picture languages. Their relation on time complexity and order is discussed. The main part is attempted to show that the language generated by a Hexagonal Sgrafitto Automata is same as the language generated by finite state automata. The deterministic cases are also discussed.

## 2. PRELIMINARIES

Here we recall the notions of hexagonal picture and hexagonal picture languages [6]. Let $\Sigma$ be a finite alphabet of symbols. A Hexagonal picture p over $\Sigma$ is a hexagonal array of symbols of $\Sigma$ and the set of all hexagonal pictures over $\Sigma$ is denoted by $\Sigma^{* * H}$.

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Example: 2.1 A Hexagonal picture over the alphabet $\{a, b, c, d\}$ is shown in figure.


For $l, m, n \geq 1, \Sigma^{l, m, n}$ denote the set of hexagonal pictures of size (l, m,n) refer [10].
Definition: 2.1 Let $\mathrm{p} \in \Sigma^{* * \mathrm{H}}$ when $\# \notin \Sigma$ is added as boundary to picture p we get a bordered version of p say $\hat{p}$. The size of $\hat{p}$ is $(l+1, m+1, n+1)$ if the size of $p$ is $(l, m, n)$. That is for every $\mathrm{p} \in \Sigma^{l, m, n}$ then $\hat{p} \in \Sigma^{l+1, m+1, n+1}$.

Example: 2.2 A hexagonal picture over the alphabet $\{a, b, c, d\}$ surrounded by \# is shown in figure.


Definition: 2.2 A pixel is an element $\mathrm{p}(i, j, k)$ of p refer [9]. We call ( $\mathrm{i}, j, k$ ) as the position of the pixel in p . We can use the term picture domain to refer the set of all possible positions in a picture of size $(l, m, n)$ without considering the borders. That is the set $l \times m \times n=\{1,2 \ldots l\} \times\{1,2 \ldots m\} \times\{1,2 \ldots n\}$. Each position has six edges. An edge for each position is identified by the adjacent positions namely U, L, LL, LR, UR, and UL. or N, NE, NW, S, SE, SW respectively.

Definition: 2.3 Let $\mathrm{p} \in \sum^{* * \mathrm{H}}$ is a hexagonal picture. The projection by mapping $\pi$ of a picture p is the picture $\mathrm{p}^{\prime} \in \sum^{* * H}$ such that $\mathrm{p}^{\prime}(\mathrm{i}, \mathrm{j}, \mathrm{k})=\pi[\mathrm{p}(\mathrm{i}, \mathrm{j}, \mathrm{k})]$ for all $1 \leq \mathrm{i} \leq \mathrm{l}, 1 \leq \mathrm{j} \leq \mathrm{m}, 1 \leq \mathrm{k} \leq \mathrm{n}$, where ( $l, m, n$ ) are the size of the hexagonal pictures.

Definition: 2.4 A hexagonal language $\mathrm{L} \subseteq \sum^{* * H}$ is recognizable if there exist a local language $L^{\prime}$ over an alphabet $\Gamma$ and a mapping $\pi$ : $\Gamma \rightarrow \sum$ such that $L=\pi\left(L^{\prime}\right)$. A hexagonal picture language $\mathrm{L} \subseteq \sum^{* * H}$ is tiling recognizable if there exist a tiling system $T=\left(\sum, \Gamma, \pi, \theta\right)$ such that $L=\pi[L(\theta)]$.

Definition: 2.5 A tiling system T is a quadruple $(\Sigma, \Gamma, \pi, \theta)$ where $\sum$ and $I$ are two finite set of symbols $\pi$ : $\Gamma \rightarrow I$ is a projection and $\theta$ is the set of tiles over the alphabet $\mathrm{I} \cup\{\#\}$.

Definition: 2.6 A non deterministic (deterministic) hexagonal on-line tessellation automata referred as 6H0TA (6-HDOTA) is defined by $A=\left(\sum, Q, q_{o}, F, \delta\right)$ where

- $\quad \sum$ is the input alphabet.
- $\quad \mathrm{Q}$ is a finite set of states.
- $\quad I \subseteq Q(I-\{i\}) \subseteq Q$ is the set of initial states.
- $\quad F \subseteq Q$ is the set of final or accepting states.
- $\quad \delta: Q \times Q \times Q \times \sum \rightarrow 2^{Q}\left(\delta: Q \times Q \times Q \times \sum \rightarrow Q\right)$ is the transition function.

In addition to this we introduce clockwise rotation of $120^{0}$ say $\mathrm{P}^{\mathrm{R}}$, vertical mirroring $\mathrm{P}^{\mathrm{VM}}$, and horizontal mirroring $\mathrm{P}^{\mathrm{HM}}$ as follows.





Remark: 2.1 By using this rotation the vertical mirroring can be expressed as horizontal mirroring and rotation. That is $\mathrm{P}^{\mathrm{VM}}=\left(\left(\mathrm{P}^{\mathrm{HM}}\right)^{\mathrm{R}}\right)^{\mathrm{R}}$.

## 3. HEXAGONAL SGRAFITTO AUTOMATA

$\mathrm{H}=\{\mathrm{E}, \mathrm{W}, \mathrm{NW}, \mathrm{NE}, \mathrm{SE}, \mathrm{SW}, \mathrm{X}\}$ is a 7 - tuple be the movement of heads in a titled picture. The first 6- tuples denote the directions, east, west, northwest, northeast, southeast, and X denote the zero movement or no movement. That is X $\in$ \#. Define a mapping V: $\mathrm{S} \rightarrow \mathrm{H}$ such that
$V(\#)=\left\{\begin{array}{l}\# \mathrm{i}=1, \mathrm{j}=1,1 \leq \mathrm{k} \leq \mathrm{n}+1 \\ \neq 1 \leq \mathrm{i} \leq 1+1, \mathrm{j}=1, \mathrm{k}=1 \\ 1 \\ \mathrm{i} \quad \mathrm{i} \leq 1,1 \leq \mathrm{j} \leq m, 1 \leq \mathrm{k} \leq \mathrm{n} \\ \mathrm{i}=\mathrm{l}+1, \mathrm{j}=\mathrm{m}+1, \mathrm{k}=\mathrm{n}+1\end{array}\right.$
The above mapping can also be defined in the planes $\mathrm{XY}, \mathrm{YZ}$ and XZ as follows.

| In XY plane | In YZ Plane | In XZ Plane |
| :--- | :--- | :--- |
| $\mathrm{a}(-\mathrm{y})=\mathrm{SE}$ | $\mathrm{a}(\mathrm{y})=\mathrm{NW}$ | $\mathrm{a}(\mathrm{z})=\mathrm{E}$ |
| $\mathrm{a}(\mathrm{y})=\mathrm{NW}$ | $\mathrm{a}(-\mathrm{y})=\mathrm{SE}$ | $\mathrm{a}(-\mathrm{z})=\mathrm{W}$ |
| $\mathrm{a}(\mathrm{x})=\mathrm{SW}$ | $\mathrm{a}(\mathrm{z})=\mathrm{E}$ | $\mathrm{a}(\mathrm{x})=\mathrm{NE}$ |
| $\mathrm{a}(-\mathrm{x})=\mathrm{NE}$ | $\mathrm{a}(-\mathrm{z})=\mathrm{W}$ | $\mathrm{a}(-\mathrm{x})=\mathrm{SW}$ |

Definition: 3.1 A 3-tape Automaton, 3-TA is defined as follows: $\mathrm{A}=\left(\mathrm{Q}, \Sigma, \Gamma, \delta, \mathrm{q}_{0}, \mathrm{Q}_{\mathrm{F}}\right)$ where,
Q- is the finite set of states.
$\Sigma$ - is the input symbol.
$\Gamma$-is the working alphabet.
$\delta$-Transition relation $\delta: \mathrm{Q} \times \Gamma^{3} \rightarrow \mathrm{Q} \times \Gamma^{3} \times \mathrm{d}^{3}$ is a transition relation
$\mathrm{q}_{0}$ - Initial state.
$\mathrm{Q}_{\mathrm{F}^{-}}$Final state
For any pair $\left(q, a_{1}, a_{2}, a_{3}\right) \in Q \times \Gamma^{3}$ each element $\left[q^{\prime},\left(a_{1}{ }^{\prime}, d_{1}\right),\left(a_{2}{ }^{\prime}, d_{2}\right),\left(a_{3}{ }^{\prime}, d_{3}\right)\right] \in \delta\left(q, a_{1}, a_{2}, a_{3}\right)$ satisfies $a_{i} \in S=>d_{i}=V\left(a_{i}\right)$, and $a_{i}=a_{i}^{\prime}$ for all $i=1,2,3 \ldots$
$\mathrm{a}_{\mathrm{i}} \notin S=>\mathrm{a}_{\mathrm{i}}{ }^{\prime} \notin S$.
Let $\mathrm{p} \in \sum^{* * \mathrm{H}}$ be an input to a bounded 3- TA, then the picture is scanned as follows.
$\boldsymbol{P}$ is the bordered picture and $\mathrm{q}_{0}$ is the initial state from where the head scans it changes to the next position as shown in the figure.

Tape-1 In X- Direction


Tape-2 In Y- Direction


Tape-3 In Z- Direction


The head scans the 3 tapes at the same time and read as $(\mathrm{a}, \varepsilon, \varepsilon),(\varepsilon, \mathrm{b}, \varepsilon),(\varepsilon, \varepsilon, \mathrm{c})$. So we can obtain (a, b, c). Then the machine accepts $p$ if and only if there is a computation starting from the initial configuration and ending in the accepting state.

Definition: 3.2 A time complexity function $\mu_{H}$ is a function $t: N \times N \times N \rightarrow N$ defined by $t(l, m, n)=m a x ~ m \in \sum 1, m, n t(p)$. $t(p)$ is the number of steps taken for a longest computation over $p$.

Definition: 3.3 Hexagonal Sgrafitto Automaton 3HSA is a tuple $A=\left(Q, \Sigma, \Gamma, \delta, q_{o}, Q_{F}, \mu\right)$ where

- $\quad\left(\mathrm{Q}, \sum, \Gamma, \delta, \mathrm{q}_{\mathrm{o}} \mathrm{Q}_{\mathrm{F}}, \mu\right)$ is a 3TA.
- $\mu: \Gamma \rightarrow N$ is a weight function or scanning function in the domain $l \times m \times n$.

The transition function satisfies $\left(\mathrm{q}^{\prime}, \mathrm{a}^{\prime}, \mathrm{b}^{\prime}, \mathrm{c}^{\prime}, \mathrm{d}\right) \in \delta(\mathrm{q}, \mathrm{a}, \mathrm{b}, \mathrm{c})$ for all $\mathrm{a} \notin S$.

$$
\mu\left(\mathrm{a}^{\prime}\right)<\mu(\mathrm{a}) \text { or } \mu\left(\mathrm{b}^{\prime}\right)<\mu(\mathrm{b}) \text { or } \mu\left(\mathrm{c}^{\prime}\right)<\mu \text { (c) for all } \mathrm{q}, \mathrm{q}^{\prime} \in \mathrm{Q} \mathrm{~d} \in \mathrm{H}, \mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{a}^{\prime}, \mathrm{b}^{\prime}, \mathrm{c}^{\prime} \in \Gamma \cup S
$$

A is a deterministic 3HSA if the 3TA is deterministic.
Lemma: 3.1 Let $A=\left(Q, \sum, \Gamma, \delta, \mathrm{q}_{\mathrm{o}} \mathrm{Q}_{\mathrm{F}}, \mu\right)$ be a 3HSA and let p be anon-empty regular hexagon of size $(\mathrm{l}, \mathrm{l}, \mathrm{l})$ then the time complexity is equal to the order of the picture.

Proof: Let p be a non-empty picture of size l. While computing over the picture pof size lit passes along z direction towards right. After reaching the end it travels one unit in y direction and scans the picture towards left and one unit in x direction and travels towards right and so on. During this scanning strategy it reduces its weight. Hence at least one of the three consecutive steps decreases weight of some field. There are be $31(l-1)+1$ fields in the picture $p$ then the weight can be decreased at most $(|\Gamma|-1)$ times. Therefore [3l (l-1) +1] $(|\Gamma|-1)$ can be performed by the scanning strategies. This will be the longest computation over $p$ which is equal to the order of the picture.

Remark: 3.1 For a hexagonal picture of size $l$, $m$, $n$ the position visited is $[l m+m n+\ln -(l+m+n)+1]$. The above result can be proved for this result also.

Lemma: 3.2 Let $\mu_{\mathrm{H}}=\left(\mathrm{Q}, \sum, \Gamma, \delta, \mathrm{q}_{\mathrm{o}} \mathrm{Q}_{\mathrm{F}}\right.$, $)$ be a 3TA. Let $\mathrm{i}, \mathrm{j}, \mathrm{k} \in \mathrm{N}$ be an integer such that during each computation of M over any picture $\sum^{* * H}$ each tape field is visited at most $i$ times along X - axis, j times along Y -axis and k times along Z-axis. Then there is a 3HSA, A such that $\mathrm{L}(\mathrm{A})=\mathrm{L}\left(\mu_{\mathrm{H}}\right)$. Moreover $\mu_{\mathrm{H}}$ is deterministic, A is also deterministic.

Proof: Define $A=\left(\mathrm{Q}, \sum, \Gamma_{1}, \delta_{1}, \mathrm{q}_{\mathrm{o}} \mathrm{Q}_{\mathrm{F}}\right.$, ) where ${ }_{1}=\sum \cup \Gamma \times\{1,2, \ldots, \mathrm{i}\} \times\{1,2, \ldots, \mathrm{j}\} \times\{1,2, \ldots, \mathrm{k}\}$ and instructions $(\mathrm{q}, \mathrm{a}, \mathrm{b}, \mathrm{c}) \rightarrow\left\{\mathrm{q}^{\prime}, \mathrm{a}^{\prime}, \mathrm{b}^{\prime}, \mathrm{c}^{\prime}, \mathrm{d}\right\}$ from $\delta$ is represented in $\delta_{1}$ by the following set of instructions. $(q, a, b, c) \rightarrow\left\{q^{\prime}, a^{\prime}, b^{\prime}, c^{\prime}, d\right\}$

Along X- Direction
$(q(a, i),(b, j),(c, z)) \rightarrow\left(q^{\prime},(a, i+1),(b, j),(c, z), d\right)$
Along Y-Direction
( $q(a, i),(b, j),(c, z)) \rightarrow\left(q^{\prime},(a, i),(b, j+1),(c, z), d\right)$
Along Z-Direction
( $q(a, i),(b, j),(c, z)) \rightarrow\left(q^{\prime},(a, i),(b, j),(c, z+1), d\right)$ for all $i=1,2 \ldots l, j=1,2 \ldots m, k=1,2 \ldots n$
Finally $\mu(\mathrm{a})=\mathrm{i} \mu(\mathrm{b})=\mathrm{j}, \mu(\mathrm{c})=$ kfor all $\mathrm{a}, \mathrm{b}, \mathrm{c} \in \sum$.
$\mu(a, i)=l-1, \mu(b, j)=m-1, \mu(c, z)=n-1$, for all $(a, i) \in \Gamma \times(1,2 \ldots l),(b, i) \in \Gamma \times(1,2 \ldots m),(c, i) \in \Gamma \times(1,2 \ldots n)$
Therefore it is a 3HSA. Clearly $L(A)=L(M)$. If $M$ is deterministic then $\delta$ is a deterministic transition relation. It follows that $\delta_{1}$ is also a deterministic transition relation. Therefore A is also deterministic.

Theorem: 3.1 Let $A=\left(Q, \Sigma, \Gamma, \delta, q_{o} Q_{F}, \mu\right)$ be a one symbol HSA. There is a finite state automaton $A_{1}$ such that $\mathrm{L}\left(\mathrm{A}_{1}\right)=\mathrm{L}(\mathrm{A})$.

Proof: Without loss of generality we assume that A moves its head while performing an instruction and it finishes its movement at $\mathrm{Q}_{\mathrm{F}}$. The proof is based on directed multigraph. The edges are labeled by the automaton. Let E be the set of labeled edges in a graph defined by $E \subseteq q \times \sum \times q$. Any edge $\left(q_{1}, a, q_{2}\right) \in E$ can be written asq $q_{1} \rightarrow q_{2}$ along $\left(q_{1}, a\right)$.In initial configuration a 3TA in normal form at any step changes its reading either by a letter of first tape or a letter of second tape or a letter of third tape. A triplet reads all the three tapes if it is accepted. We can describe a successful path using a hexagonal picture as follows. In3TA the edges are as follows: $(\mathrm{x}, \varepsilon, \varepsilon),(\varepsilon, \mathrm{y}, \varepsilon),(\varepsilon, \varepsilon, \mathrm{z})$.

That is $\mathrm{E} \subseteq(\mathrm{q} \times(\mathrm{x}, \varepsilon, \varepsilon) \times \mathrm{q}) \quad \mathrm{U}(\mathrm{q} \times(\varepsilon, \mathrm{y}, \varepsilon) \times \mathrm{q}) \quad \mathrm{U}(\mathrm{q} \times(\varepsilon, \varepsilon, \mathrm{z}) \times \mathrm{q})$. Consider hexagonal picture of size $(\mathrm{l}, \mathrm{m}, \mathrm{n})$. There arise three cases.

## Case: 1

For $\mathrm{q}_{1} \rightarrow \mathrm{q}_{2}$ 3TA goes from ( $\mathrm{i}, \mathrm{j}, \mathrm{k}$ ) to ( $\mathrm{i}^{\prime}, \mathrm{j}^{\prime}, \mathrm{k}^{\prime}$ )

1. $\mathrm{k}=0$ then $\left(\mathrm{i}^{\prime}, \mathrm{j}^{\prime}, \mathrm{k}^{\prime}\right)=(\mathrm{i}+1, \mathrm{j}, 0)$
2. $k \neq 0, i=0, j \neq m$ then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(0, j+1, k-1)$
3. $k \neq 0, i=0, j=m$ then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(1, j, k)$
4. $k \neq 0, I \neq 0$, then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(i+1, j, k)$

Case: 2

1. $\mathrm{j}=\mathrm{m}$ then $\left(\mathrm{i}^{\prime}, \mathrm{j}^{\prime}, \mathrm{k}^{\prime}\right)=(\mathrm{i}+1, \mathrm{~m}, \mathrm{k}+1)$
2. $j \neq m$ then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(i, j+1, k)$

Case: 3

1. $\mathrm{i}=0$ then $\left(\mathrm{i}^{\prime}, \mathrm{j}^{\prime}, \mathrm{k}^{\prime}\right)=(0, \mathrm{j}, \mathrm{k}+1)$
2. $i \neq 0, k=0, j \neq m$ then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(i-1, j+1,0)$
3. $i \neq 0, k \neq 0, j=m$ then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(i, m, k+1)$
4. $k \neq 0, I \neq 0$, then $\left(i^{\prime}, j^{\prime}, k^{\prime}\right)=(i, j, k+1)$

By using these cases any string in A can be formed as follows. Consider $x^{3} y^{3} z^{3}$. Let the states be $q_{0}, q_{1}, q_{2}, q_{3}$. Then $E=\left\{\left(\mathrm{q}_{0},(\mathrm{a}, \varepsilon, \varepsilon), \mathrm{q}_{1}\right),\left(\mathrm{q}_{1},(\varepsilon, \mathrm{~b}, \varepsilon), \mathrm{q}_{2}\right),\left(\mathrm{q}_{2},(\varepsilon, \varepsilon, \mathrm{c}), \mathrm{q}_{3}\right),\left(\mathrm{q}_{3},(\mathrm{a}, \varepsilon, \varepsilon), \mathrm{q}_{1}\right)\right\}$. The language for $\mathrm{x}^{3} \mathrm{y}^{3} \mathrm{z}^{3}$ is

$$
\left(\mathrm{q}_{0},(\mathrm{a}, \varepsilon, \varepsilon), \mathrm{q}_{1}\right) \rightarrow\left(\mathrm{q}_{1,},(\varepsilon, \mathrm{~b}, \varepsilon), \mathrm{q}_{2}\right) \rightarrow\left(\mathrm{q}_{2},(\varepsilon, \varepsilon, \mathrm{c}), \mathrm{q}_{3}\right)
$$

$\rightarrow\left(\mathrm{q}_{3},(\mathrm{a}, \varepsilon, \varepsilon), \mathrm{q}_{1}\right) \rightarrow\left(\mathrm{q}_{1},(\varepsilon, \mathrm{~b}, \varepsilon), \mathrm{q}_{2}\right) \rightarrow\left(\mathrm{q}_{2},(\varepsilon, \varepsilon, \mathrm{c}), \mathrm{q}_{3}\right)$
$\rightarrow\left(\mathrm{q}_{3},(\mathrm{a}, \varepsilon, \varepsilon), \mathrm{q}_{1}\right) \rightarrow\left(\mathrm{q}_{1},(\varepsilon, \mathrm{~b}, \varepsilon), \mathrm{q}_{2}\right) \rightarrow\left(\mathrm{q}_{2},(\varepsilon, \varepsilon, \mathrm{c}), \mathrm{q}_{3}\right)$
The graph representation is given below


Fig.4.
Here $\left\{\mathrm{q}_{0}, \mathrm{q}_{1}, \mathrm{q}_{2}, \mathrm{q}_{3}\right\}$ represents the repeating vertex sets and $\left(\mathrm{q}_{0}, \mathrm{q}_{1}\right)$, $\left(\mathrm{q}_{1}, \mathrm{q}_{2}\right),\left(\mathrm{q}_{2}, \mathrm{q}_{3}\right)$ and $\left(\mathrm{q}_{3}, \mathrm{q}_{1}\right)$ represents the repeating edges. Now it remains to check whether $A$ accepts the automaton .If the vertices are renamed as $f_{0}, f_{1}, \ldots, f_{n+1}$. Then we have to show that there is a path from $f_{1}, \ldots, f_{n}$ exist excluding the beginning and ending node. Consider an edge incoming to $f_{1}$ from the $X, Y$ or $Z$ direction. It goes to $Y, Z$ or $X$ direction respectively. Same argument can be shown for outgoing edges also. Let $E_{i}$ be the sequence of outgoing and incoming edges from and to $f_{i}$ then for $f_{k}$ $2 \leq k \leq n-1 . E_{k}=\left\{e_{1}, e_{2} \ldots e_{s}\right\}$ then graph is accepted if

1. $\mathrm{S} \geq 3$ and it is an odd number for n is odd even number for n is even.
2. If $e_{j}$ is an incoming edge to $f_{k}$ then $j$ is odd otherwise $j$ is even.
3. $\quad e_{j}$ is incoming to $f_{k}$ from $X$ direction then $e_{s}$ will be an outgoing edge from $f_{k} Y$-direction , $e_{j}$ is incoming to $f_{k}$ from $Y$ direction then $e_{s}$ will be an outgoing edge from $f_{k} Z$-direction also if $e_{j}$ is incoming to $f_{k}$ from $Z$ direction then $e_{s}$ will be an outgoing edge from $f_{k} X$-direction.
4. For each $j<s e_{j}$ will be an outgoing in $X, Y$ or $Z$ - direction then $e_{j-1}$ will be incoming edge from $Y, Z, X$ direction. Similar condition holds for all $k$. Therefore constructing direct path starting from $f_{1}$ we get a path ending in $f_{n}$. To show the above constructed path accepts A.

Consider a vertex $f_{i}$, and an outgoing edge from $f_{i}$ of index say $j$ that is ej. The edge labeled $\left(q_{1}, a\right) \rightarrow\left(q_{1}, a, d\right)$.If $j=1$ that is only one edge then $f_{i}=f_{1}$ and $\left(q_{1}, a\right)$ reduces to ( $\left.q_{0}, a\right)$. Otherwise there will be one or more edge which is incoming from the edge index $\mathrm{j}-1$. That is not possible. Therefore for $\mathrm{j}>2$ there is an outgoing as well as an incoming edge named $q_{1}$ and $q_{3}$. Proceeding like this for $j=n$ we get $f_{n}$ as last incoming edge. That is it ends at the accepting state. Therefore A is an accepting automaton.

## Anitha. $P^{* 1}$ and Dersanambika. K. $S^{2}$ / Hexagonal Sgrafitto Automata / IRJPA- 4(4), April-2014.

By using this accepting computation on A we can non-deterministically guess a multigraph representing this accepting computation. This gives us a nondeterministic finite state automaton $A_{1}$. As $A_{1}$ moves the head to the nodes $f_{i}$ it fulfills the required properties. Since the incoming and outgoing edge satisfies the properties it is also accepted. Therefore $\mathrm{L}(\mathrm{A})=\mathrm{L}\left(\mathrm{A}_{1}\right)$.

## 4. CONCLUSION

We have introduced a model of automaton called hexagonal sgrafitto automata. An attempt is made to extend the result in two dimension two three dimensional space. The accepting states of this automaton are also discussed. Further researches can be done on time complexity.

## REFERENCES

1. Daniel Prusa, Frantisek Marz, New models for Recognition of picture languages: Sgrafitto and restarting tiling automata.CTU-CMP-2012-08, march20, 2012.
2. Dora Giammarresi, Antonio Restivo, Two dimensional languages in G.Rozenberget. al. (eds) in Hand book of formal language Vol.III, Springer Verlag, 1997, pp.215-268
3. D. Giammarressi and A. Restivo, Two dimensional finite state recognizability. Fundamentainformatical 25 (3, 4): 399- 422. 1966
4. G.Siromoney and R.Siromoney, Hexagonal arrays and rectangular blocks computer graphics and Image Processing, 5:353-381, 1976
5. K.Inoue and A.Nakamura, Some properties of two dimensional ontine tessilation acceptor In Form Sci. (13) 1997, 95-121.
6. K.S.Dersanambika and K.Krithivasan, C.Martin - vide, K.G. Subramanian, Hexagonal Pattern Languages. IWCIA 2004, LNCS 3322, 2004, 52-64.
7. K.S.Dersanambika and K.Krithivasan, C.Martin-vide, K.G. Subramanian, Local and Recognizable Hexagonal picture languages. International journal of pattern recognition and Artificial intelligence.19, (7), 2005, 853-871
8. M. Anselmo, D. Giammarresi, and M. Madonia. Tiling automaton: A computational model for recognizable twodimensional languages. In Proc. CIAA 2007, 4783, lecture notes on computer science, Springer 2007. 290-302
9. M.Anselmo, M.Madonia, Deterministic and unambiguous two dimensional languages over one letter alphabet. Theoretical computer science. 410 (2009:1477-1485)
10. M.Blum and C.Hewitt Automata on a two dimensional tape. IEEE Symposium on Switching and Automata theory, pages 155-160, 1967
11 P.Anitha, K. SujathaKumari, and K.S. Dersanambika. Hexagonal picture languages and recognizability. Journal of Science and Technology and management, (4), 2011, 17-26.

## Source of Support: Nil, Conflict of interest: None Declared

