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LEFT REVERSE DERIVATIONS ON SEMIPRIME RINGS

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ABSTRACT

In this paper some results concerning to left reverse derivations on semi prime rings are presented. A mapping d on a semi prime ring R is a left reverse derivation if and only if it is a central derivation.

Key words: *Prime ring, Semi prime ring, derivation, Reverse derivation, Central derivation.*

INTRODUCTION

Bresar and Vukman [1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [2]. Macdonald [3] established some group-theoretic results in terms of inner derivations. Bell and Kappe [4] studied the analogous results for rings in which derivations satisfy certain algebraic conditions. In this paper, we study these results for the rings with left reverse derivations.

PRELIMINARIES

Throughout, R will represent a semi prime ring. We recall that a ring R is called prime if $aRb=0$ implies $a=0$ or $b=0$; and it is called semi prime if $aRa=0$ implies $a=0$. An additive mapping d from R into itself is called a derivation if $d(xy)=d(x)y+xd(y)$ for all $x,y\in R$ and is called a left reverse derivation if $d(xy)=yd(x)+xd(y)$ for all $x,y\in R$. A mapping d from R into itself is called central derivation if $d(x)\in Z$ for all x in R.

MAIN RESULTS

Theorem: 1 A mapping d on a semi prime ring R is a left reverse derivation if and only if it is a central derivation.

Proof: Let R be a semi prime ring and d: R $\rightarrow$ R a mapping on R. It is clear that if d is a central derivation, then d is a left reverse derivation. So, let us suppose that d is a left reverse derivation, then

$d(xy^2) = y^2 d(x) + x d(y^2)$

$\quad = y^2 d(x) + x d(yy)$

$\quad = y^2 d(x) + x(yd(y) + yd(y))$

$$d(xy^2) = y^2 d(x) + xy d(y) + xy d(y), \text{ for all } x, y \in R \tag{1}$$

Also, $d((xy)y) = yd(xy) + xy d(y)$

$\quad = y(yd(x) + x d(y)) + xy d(y)$

$$\therefore d(xy^2) = d((xy)y) = y^2 d(x) + yx d(y) + xy d(y), \text{ for all } x, y \in R \tag{2}$$

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From equ's (1) and (2), we have,

$\Rightarrow$ $y^2d(x) + xy\, d(y) + xy\, d(y) = y^2d(x) + yx\, d(y) + xy\, d(y)$

$\Rightarrow$ $xy\, d(y) = yx\, d(y)$

$\Rightarrow$ $yx\, d(y) - xy\, d(y) = 0$

$\Rightarrow$ $(yx – xy)d(y) = 0$

$\Rightarrow$ $[y, x]d(y) = 0$, for all $x, y \in R$ (3)

We replace x by zx in equ. (3), and using (3) again, we get,

$\Rightarrow$ $[y, zx]\, d(y) = 0$

$\Rightarrow$ $(z[y, x] + [y, z]x)d(y) = 0$

$\Rightarrow$ $z[y, x]d(y) + [y, z]x\, d(y) = 0$

$\Rightarrow$ $[y, z]x\, d(y) = 0$

By interchanging x and z in the above equation, we get,

$\Rightarrow$ $[y, x]z\, d(y) = 0$, for all x, y, z in R (4)

On the other hand, a linearization of equation (3) leads to,

$\Rightarrow$ $[y + u, x]d(y +u) = 0$

$\Rightarrow$ $((y + u)x – x(y + u))(d(y) + d(u)) = 0$

$\Rightarrow$ $(yx + ux - xy – xu)(d(y) + d(u)) = 0$

$\Rightarrow$ $(yx – xy + ux – xu)(d(y) + d(u)) = 0$

$\Rightarrow$ $([y, x] + [u, x])(d(y) + d(u)) = 0$

$\Rightarrow$ $[y, x]d(y) + [u, x]d(y) + [y, x]d(u) + [u, x]d(u) = 0$

$\Rightarrow$ $[y, x]d(u) + [u, x]d(y) = 0$

$\Rightarrow$ $[u, x]d(y) = - [y, x]d(u)$ (5)

$\Rightarrow$ $[u, x]d(y) = [x, y]d(u)$

We replace z by d(u)z[u, x] in equation (4), then we get,

$\Rightarrow$ $[y, x]\, d(u)z[u, x]d(y) = 0$

$\Rightarrow$ $- [y, x]\, d(u)z[y, x]d(u) = 0$

$\Rightarrow$ $[y, x]\, d(u)z[y, x]d(u) = 0$ (6)

Since R is semi prime, by equation (6) we get, $[y, x]d(u) = 0$, for all $x, y, u \in R$. By [5], $d(u) \in z$ for all $u \in R$. Hence, $d(xy) = yd(x) + xd(y)$. This shows that d is a left reverse derivation on R which maps R into its center.

As a consequence, we get the following

Corollary: 1 Let R be a prime ring. If R admits a non- zero left reverse derivation d, then R is commutative.

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