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UNION FUZZYSOFT N-GROUP

A. Solairaju[1], P. Sarangapani[2] and R. Nagarajan[3*]

[1]AssociateProfessor, PG & Research Department of Mathematics, Jamal Mohamed college, Triuchirappalli-20, Tamilnadu, India

[2]Assistant Professor, Department of Computer Science, Kurinji College of Arts & Science, Tiruchirappalli-02, Tamilnadu, India

[3]Associate Professor, Department of Mathematics, J J College of Engineering & Technology, Tiruchirappalli-09, Tamilnadu, India

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ABSTRACT

In this paper, we introduce Union Fuzzy soft N-Group by using Molodtsov's definition of soft sets and investigate their related properties with respect to α-inclusion of soft sets.

Keywords: *Soft set – Fuzzy Soft set – Soft N-group – Union Fuzzy Soft N-Group - α inclusion.*

SECTION - 1:

INTRODUCTION

In 1999, Molodtsov's [26] proposed an approach for Modeling, Vagueness and Uncertainity, called soft set theory, since its inception, works on soft set theory have been progressing rapidly with a wide range applications especially in the mean of Algebraic structures as in [2-12]. The structures of soft sets operations of soft sets and some related concepts have been studied by [14-19]. The theory of soft set continues to experience tremendous growth and diversification in the mean of soft decision making as in the following studies [20-23] as well. Atagun and Sezgin [33] defined soft N-subgroups and soft N-ideals of an N-group, they studied their properties with respect to soft set operators in more detail. In this paper we introduce Union Fuzzy Soft N-Group by using Molodtsov's definition of soft sets and investigate their related properties with respect to α -inclusion of soft sets.

SECTION - 2: PRELIMINARIES

Definition 2.1: Let $(\Gamma, +)$ be a group and μ: N X $\Gamma\to\Gamma(n, \nu)\to n\nu$, (Γ, μ) is called an N-group if x, y$\in$ N and $\forall\nu\in\Gamma$,
(i) $x(y\nu)=(xy)\nu$ and
(ii) $(x+y)\nu = x\nu + y\nu$. It is denoted by N^{Γ}.

Clearly N itself is an N-group by natural operation. A subgroup H of $\Gamma$with NH$\subseteq$ H is said to be an N-subgroup of $\subseteq\Gamma$. Γandψ be two N- groups then f:$\Gamma\to\psi$ is called an N-homomorphism if $\forall\nu$, H $\in\Gamma$, $\forall$ n $\in$ N
(i) $f(\nu+H) = f(\nu)+f(H)$ and
(ii) $f(n\nu) = nf(\nu)$

For all undefined concepts and notations, we refer to [29]. From now on U refers to initial universe, E is a set of parameters $2^U$is the power set of U and A, B, C $\subseteq$ E

Definition 2.2: Let U be any Universal set, E set of parameters and A$\subseteq$ E, then a pair (F,A) is called soft set over U, where F is a mapping from A to 2^U , the power set of U.

Example 2.1: Let X={c_1,c_2,c_3} be the set of three cars and E={costly(e_1), metallic colour (e_2), cheap(e_3)} be the set of parameters, where A={e_1,e_2} $\subset$ E. then (F,A)={F(e_1)={c_1,c_2,c_3},F(e_2)={c_1,c_2} } is the crisp soft set over X.

***Corresponding author:** *R. Nagarajan[3*]*

Definition 2.3: Let U be the universal set, E set of parameters and $A \subset E$. Let F(X) denote the set of all fuzzy subsets of U, then a pair (F,A) is called fuzzy soft set over U, where F is a mapping from A to F(U).

Example 2.2: Let $U=\{c_1,c_2,c_3\}$ be the set of three cars and $E=\{costly(e_1), metalliccolor(e_2), cheap(e_3)\}$ be the set of parameters, where $A=\{e_1,e_2\} \subset E$. then $(F,A)=\{F(e_1)=\{c_1/0.6, c_2/0.4, c_3/0.3\}, F(e_2)=\{c_1/0.5, c_2/0.7, c_3/0.8\}$ is the fuzzy soft set over U denoted by F_A.

Definition 2.4: Let F_A be a fuzzy soft set over U and α be a subset of U then upperα - inclusion of $F_A$denoted by $F^{\alpha}{}_A = \{x \in A \,/ F(x) \geq \alpha\}$. Similarly $F^{\alpha}{}_A = \{x \in A \,/\, F(x) \leq \alpha\}$ is called lower α-inclusion of F_A.

Definition 2.5: Let F_A and G_B be fuzzy soft sets over the common universe U and ψ: $A \rightarrow B$ be a function then fuzzy soft image of F_A under ψ over U denoted by $\psi(F_A)$ is a set-valued function, where $\psi(F_A)$: $B \rightarrow 2^U$ defined by

$$\psi(F_A)\,(b)=\{\cup\{F(a) \,/\, a \in A \text{ and } \psi\,(a)=b\}$$

If $\psi^{-1}(b) \neq \varphi\}$for all $b \in B$, the soft pre-image of G_B under ψ over U denoted by $\psi^{-1(}G_B)$ is a set-valued function, where $\psi^{-1(}G_B)$: $A \rightarrow 2^U$ defined by $\psi^{-1}(G_B)(b) = G(\psi(a))$ for all $a \in A$ then fuzzy soft anti-image of F_A under ψ over U denoted by $\psi(F_A)$ is a set-valued function, where $\psi(F_A)$:$B \rightarrow 2^U$ defined by $\psi^{-1}(F_A)(b)=\{\cap\{F(a) \,/\, a \in A$ and $\psi\,(a)=b\}$, if $\psi^{-1}(b) \neq \varphi$ for all $b \in B$

Definition 2.6: Let H be an N-subgroup of Γ and F_H be a fuzzy soft over Γ. If for all $x, y \in H$ and $n \in N$,
(i) $F(x\text{-}y) \leq F(x) \cup F(y)$ and
(ii) $F(nx) \leq F(x)$ then the fuzzy soft set F_H is called a union fuzzy soft N-subgroup of Γ and denoted by $F_H <_N \Gamma$

Example 2.3: Consider N= {0, 1, 2, 3} be a group with operation +

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

If we define a fuzzy soft set G_H over Γ by

$G(x) =\{y \in \Gamma / 3x=y\}$ for all $x \in H$.

Then G(0)={0} and G(2) ={2} since $G(2\text{-}2) = G(0) \neq G(2)$,$G_H$ is not a union soft N-subgroup of Γ

Definition 2.7: The relative complement of the fuzzy soft set F_A over U is denoted by $F^r{}_A$ where $F^r{}_A$: $A \rightarrow 2^U$ is a mapping given as $F^r{}_A(x) = U/F_A(x)$, for all $x \in A$.

SECTION - 3: CHARACTERIZATION'S OF UNION FUZZY SOFT N-GROUP

Proposition 3.1: Let F_A be a fuzzy soft set over Γ and α be a subset of Γ. If F_A is a union fuzzy soft N-subset of Γ, then upper α- inclusion of F_A is an N-subgroup of Γ.

Proof: Since F_A is union fuzzy soft N-subgroup of Γ. Assume $x, y \in F_A{}^{\alpha}$ and $n \in N$, then $F(x) \geq \alpha$ and $F(y) \geq \alpha$, we need to show that $x\text{-}y \in F_A{}^{\alpha}$and $n \in F_A{}^{\alpha}$ since F_A is union fuzzy soft N-subgroup of Γ, it follows that $F(x\text{-}y) \leq \max\{F(x), F(y)\} \geq \min\{\alpha,\alpha\} \geq \alpha$ and $F(nx) \leq \alpha \geq \alpha$ which completes the proof.

Proposition 3.2: Let F_A be a fuzzy soft set over Γthen F_A is a union fuzzy soft N-subgroup of Γ if $F_A{}^r$ is fuzzy soft N-subgroup of Γ.

Proof: Let F_A be a union fuzzy soft N-subgroup of Γ.Then for all $x, y \in A$ and $n \in N$.

$$\begin{aligned} F_A{}^r(x\text{-}y) &= \Gamma \,/\, F_A(x\text{-}y) \\ &\geq \Gamma / \max\{F_A(x),F_A(y)\} \\ &= \min\{(\Gamma \,/\, F_A\,(x)),\ \Gamma / F_A(y)\} \\ &= \min\{F_A{}^r(x),\ F_A{}^r(y)\} \end{aligned}$$

$$\begin{aligned} F_A{}^r(n\,x) &= \Gamma \,/\, F_A(n\,x) \\ &\geq \Gamma \,/\, F_A(x) \end{aligned}$$

$F_A{}^r$ (n x) = $F_A{}^r(x)$, $F_A{}^r$ is fuzzy soft N-subgroup ofΓ.

Proposition3.3: Let F_A: $X \rightarrow X^1$ be a soft homomorphism of N-subgroups. If F_A is union fuzzy soft N-subgroups of X^1, then F_A is union fuzzy soft N-subgroups of X^1.

Proof: Suppose F_A is union fuzzy soft N-subgroups of X^1, then
(i) Let $x^1, y^1 \in X^1$, then exists x, y $\in$X such that

$f(x)= x^1$ and $f(y)=y^1$, we have

$$F_A(x^1-y^1) = F_A(f(x)-(f(y))$$
$$\leq \max\{F_A(f(x)), F_A(f(y)\}$$

$$F_A(x^1-y^1) = \max\{F_A{}^1(x), F_A{}^1(y)\}$$

$$\text{(ii) } F_A(n\,x^1) = F_A(nf(x))$$
$$\leq F_A{}^f(x)$$

$$F_A(n\,x^1) = F_A{}^f(x)$$

.: F_A is union fuzzy soft N-subgroups X^1

Proposition3.4: Let F_A be union soft N-sub groups of X and $F_A{}^\alpha$ be a fuzzy soft set in X given by $F_A{}^\alpha(x) = F_A(x) + 1 - F(1)$ for all x $\in$X then $F_A{}^\alpha$is union fuzzy soft N-subgroups of X and $F_A \subseteq F_A{}^\alpha$.

Proof: Since F_A is union fuzzy soft N-subgroups of X and $F_A{}^\alpha(x) = F_A(x) + 1 - F_A(1)$ for x $\in$ X. For any x, y $\in$ X, we have $F_A(1)=F_A(1)+1-F_A(1)=1>F_A{}^\alpha(x)$ and for all x, y $\in$ X, we have

$$F_A{}^\alpha(x-y) = F_A(x-y)+1-F_A(1)$$
$$\leq \max\{F_A(x), F_A(y)\}+1-F_A(1)$$
$$= \max\{F_A(x)+1-F_A(1), F_A(y)+1-F_A(1)\}$$
$$= \max\{F_A{}^\alpha(x), F_A{}^\alpha(y)\}$$

$$F_A{}^\alpha(nx) = F_A(nx) +1-F_A(1)$$
$$= F_A(x) +1-F_A(1)$$
$$= F_A{}^\alpha(x),\ F_A{}^\alpha \text{ is union fuzzy soft N-subgroup of X.}$$

Proposition3.5: Let F_A and G_B to fuzzy soft gets over Γ, where A and B are N- groups of Γ and ø: A $\rightarrow$ B is an N-homomorphism. If F_A is union fuzzy soft N- subgroups of Γ, then so is ø (F_A).

Proof: Let $\alpha_1, \alpha_2 \in$ B such ø is surjective, there exists $a_1, a_2 \in A$ such that Ø (a_1) = α_1 and Ø (a_2) = α_2 thus

$$(Ø\, F_A)(\alpha_1 - \alpha_2) = \max\{F(a)/A \in A,\ Ø\,(A)=\alpha_1 - \alpha_2\}$$
$$= \max\{F(a)/A \in A, A=Ø^{-1}(\alpha_1 - \alpha_2)\}$$
$$= \max\{F(a)/A \in A, A=Ø^{-1}(ø(a_1 - a_2))= A_1 - A_2\}$$
$$= \max\{F(a_1 - a_2)/\alpha_1,\alpha_2 \in B, Ø\,(A_i)= \alpha_i, i=1,2\}$$
$$= \min\{(\max\{F(a_1)/\alpha_1 \in B, Ø\,(a_1)=\alpha_1)\},\{\max(F(a_2)/\alpha_2 \in B, Ø\,(a_2)=\alpha_2)\}$$
$$= \min\{Ø\,(F_A)(a_1), Ø\,(F_A)(a_2)\}$$

Now let n $\in$N and $\alpha \in$ B. Since Øsurjective, then exists $\bar{A} \in A$ such that Ø ($\bar{A}$) = 0

$$(Ø\,(F_A)\,(n\,\alpha) = \max\{F(A)/A \in A,\ Ø\,(A)=n\alpha\}$$
$$= \max\{F(A)/A \in A, A=Ø^{1}(n\alpha)\}$$
$$= \max\{F(A)/A \in A, A=Ø^{-1}(nø(\bar{A}))\}$$
$$= \max\{F(A)/A \in A, A=Ø^{-1}(ø(nA))=n\bar{A}\}$$
$$= \max\{F(n\bar{A})/\bar{A} \in A,\ Ø\,(\bar{A})=\alpha\}$$
$$= \max\{F(\bar{A})/\bar{A} \in A,\ Ø\,(\bar{A})=\alpha\}$$
$$= \max(Ø\,(F_A))(\alpha)$$

Ø (F_A) is union fuzzy soft N-subgroup of Γ.

Proposition3.6: Let F_A:X$\rightarrow$Y be a soft homomorphism of N-subgroups. If F_A is union fuzzy soft N-subgroups of Y, then $F_A{}^f$ is union fuzzy soft N-subgroups of X.

Proof: Suppose F_A is union fuzzy soft N-subgroups of Y, then

i) For all x, y € X, we have

$$\begin{aligned} F_A(x-y) &= F_A(f(x-y)) \\ &= F_A(f(x)-f(y)) \\ &= \max\{F_A(f(x),F_A(f(y))\} \\ &= \max\{F_A{}^f(x),F_A{}^f(y)\} \end{aligned}$$

ii) $$\begin{aligned} F_A{}^f(n\,x) &= F_A(f(n\,x)) \\ &\leq F_A(f(x)) \\ &= F_A{}^f(x) \end{aligned}$$

$F_A{}^f$ is union fuzzy soft N- subgroups of X.

Proposition3.7: Let F_A and G_B be fuzzy soft sets over Γ, where A and B are N-subgroups of Γand Ø be on N-homomorphism from A to B if G_B is a union fuzzy soft N-subgroup of Γ, then so is $Ø^{-1}(G_B)$.

Proof: Let $a_1, a_2 \in A$, then

$$\begin{aligned} (Ø^{-1}(G_B))(a_1-a_2) &= G(Ø(a_1-a_2)) \\ &\geq \max\{G(\{Ø(a_1),Ø(a_2)\} \\ &= \max\{(Ø^{-1}(G_B)(a_1),Ø^{-1}(G_B(a2))\} \end{aligned}$$

Now let $n \in N$ and $A \in A$, then

$$\begin{aligned} (Ø^{-1}(G_B))(nA) &= G(Ø(nA)) \\ &= G(n\,Ø(A)) \\ &= G(Ø(A)) \\ &= (Ø^{-1}(G_B))(A) \end{aligned}$$

$Ø^{-1}(G_B)$ is a union fuzzy soft N-subgroups of Γ.

CONCLUSION

This paper summarized the basic concepts of soft sets.By using these concepts we studied the algebraic properties of union fuzzy soft N-groups.This work focused on fuzzy soft pre-image, fuzzy soft image. fuzzy soft anti image.To extend this work one could study theproperties of fuzzy soft N-groups in other algebraic structures such as rings and fields.

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