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ON SEMI-HOMEOMORPHISMS

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ABSTRACT

Properties of semi-homeomorphisms, semi compact spaces are discussed.

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Keywords: *Semi-homeomorphism, irresolute mapping, semi pre open mapping.*

INTRODUCTION

In this section, we recall some definitions

A set A is said to be semi – open if there exists an open set G such that

$G \subset A \subset \text{cl}\, G$ or $A \subset \text{cl int}\, A$, The set A is semi-closed if $\text{int cl}\, A \subset A$

A function $f : X \to A$ is said to be irresolute if $f^{-1}(A)$is a semi-open set whenever A is a semi open set in Y. The function f is called a pre semi open function if f(A) is a semi open set whenever A is a semi – open set.

A bijection $f : X \to Y$ is a semi-homeomorphism if f is irresolute and pre-semi-open. A space X is said to be semi compact if every cover of X by semi-open sets has a finite sub-cover.

MAIN RESULTS

Theorem 1: Let $f : X \to Y$ be a bijection. Then f is semi pre open if and only if $f^{-1} : Y \to X$ is irresolute.

Proof:
Step - 1: suppose that the bijection f^{-} is semi pre open then $f^{-1\,-}$ is a function from Y to X

Let U be a semi open set in X. Then f(U) is a semi open set in Y. But f(U) = $(f^{-1})^{-1}$ (U). Hence $(f^{-1})^{-1}$ (U) is a semi open set in Y. Put $g = f^{-1}$. We have $g^{-1}(U)$ is semi open.

Consequently g is irresolute. That is, $f^{-1}$is irresolute.

Step – 2: suppose that f is irresolute let U be a semi open set in X. Then $(f^{-1})^{-1}$ (U) is a semi open set in Y. But $(f^{-1})^{-1}(U) = f(U)$ Therefore f(U) is a semi open set in Y. Hence f is pre semi open.

Theorem 2: If $f : X \to Y$ and $g : Y \to Z$ are both irresolute, than their composition $g \circ f : X \to Z$ is an irresolute map.

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Proof: Let V be and semi open set in Z. Then

$$(g \circ f)^{-1}(V) = (f^{-1} \circ g^{-1}(V)$$
$$= (f^{-1}(g^{-1}(V))$$

Since g is irresolute, it follows that $g^{-1}(V)$ is a semi open set. Since f is irresolute, it follows that $(f^{-1}(g^{-1}(V))$ is a semi open set. Thus for each semi open set V in Z, $(g \circ f)^{-1}(V)$ is semi open in X. Therefore, $g \circ f$ is an irresolute function.

Theorem 3: Semi-homeomorphism is an equivalence relation. We write X ~ Y whenever two spaces X,Y are semi-homeomorphic.

Proof:

Step – 1: Let $i: X \to X$ be the identity map on X. Then it is bijective and irresolute. Also $(i)^{-i}$ is a pre semi open map. Hence i is a semi-homeomorphism. Accordingly X ~ X. The relation is reflexive.

Step – 2: Suppose that X ~ Y. Then there exists a semi-homeomorphism. $h: X \to Y$ But then h is bijective. Accordingly $h^{-1}: Y \to X$ is bijective. Also h is irresolute. Hence h^{-1} is a pre semi open map; Hence Y ~ X.

Step – 3: Suppose that X ~ Y and Y ~ Z. Then there is a semi-homeomorphism. $f\ : X \to Y$ and there is a semi-homeomorphism g from Y to Z. But then f and g are bijective. Accordingly $g \circ f$ is bijective and pre semi open. Thus $g \circ f$ is semi-homeomorphism. Therefore X ~ Z. Hence ~ is transitive. From step (1) (2) and (3) semi-homeomorphism is an equivalence relation.

Theorem 4: Every semi- compact subset of a Hausdorff space is semiclosed.

Proof: Suppose that A be a semi compact subset of a Hausdorff space X. Let $x \varepsilon X - A$. . Then there are disjoint semi open sets U_x and V_x such that $x \varepsilon U_x$ and $A \subset V_x$. . But then $X \varepsilon U_x \subset X - V_x \subset X - A$. Therefore X-A is semi open. Hence A is semi-closed in Y.

Theorem 5: Let X be semi compact and set Y be a Hausdorff space. If $f: X \to Y$ is continuous irresolute and bijective, then f is a semi-homeomorphism.

Proof: Let A be a semi-closed subset of the semi compact space X. Then A is semi-compact. But f is irresolute. Hence f(A) is semi compact. Take $g = f^{-1}$. Then $g^{-1}(A)$ is semi closed, by theorem 3. Consequently g is an irresolute map. That is, f^{-1} is irresolute. Therefore f is a semi-homeomorphism.

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