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$(\tau_1, \tau_2)^*$ - $Q^\star$ HOMEOMORPHISM IN BITOPOLOGICAL SPACES

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ABSTRACT

Aim of this paper is to introduce and study the new type of homeomorphism, namely $(\tau_1, \tau_2)^*$ - $Q^\star$ homeomorphism and $\tau_1\tau_2$ - $Q^\star$ homeomorphism in bitopological spaces. Also we define $(\tau_1, \tau_2)^*$ -irreducible spaces and $\tau_1\tau_2$ - irreducible spaces. Here researchers proved that the set of all $(\tau_1, \tau_2)^*$ - $Q^\star$ homeomorphism forms a group.

Keywords: *$(\tau_1,\tau_2)^*$ - $Q^\star$ homeomorphism, $\tau_1\tau_2$ - $Q^\star$ homeomorphism, $(\tau_1,\tau_2)^*$ - irreducible spaces and $\tau_1\tau_2$-irreducible spaces.*

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1. INTRODUCTION

The notion of homeomorphism plays a dominant role in topology and so many authors introduced various types of homeomorphisms in topological spaces. In 1995, Maki, Devi and Balachandran [3] introduced the concepts of semi-generalized homeomorphisms and generalized semi-homeomorphisms and studied some semi topological properties. Devi and Balachandran introduced a generalization of α-homeomorphism in 2001.

Recently, P. Padma and S. Udaykumar [8] introduced and studied the concept of $(\tau_1, \tau_2)^*$- Q^* continuous maps in bitopological spaces.

The purpose of this paper is to introduce the concepts of homeomorphisms by using $(\tau_1, \tau_2)^*$ - Q^* open sets. In this paper, we introduce the concepts $\tau_1\tau_2$ - $Q^\star$ homeomorphism, and $(\tau_1, \tau_2)^*$ - $Q^\star$ homeomorphism and investigate their basic properties. Also we define and studied the properties of $(\tau_1, \tau_2)^*$ -irreducible spaces and $\tau_1\tau_2$ - irreducible spaces.

The most important property is that the set of all $(\tau_1, \tau_2)^*$ - $Q^\star$ homeomorphisms is a group under composition of functions.

2. PRELIMINARIES

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2). Now we shall require the following known definitions are prerequisites.

Definition 2.1: A subset S of X is called $(\tau_1, \tau_2)^*$-open if $S\in\tau_1 \cup \tau_2$ and the complement of $(\tau_1, \tau_2)^*$ - open set is $(\tau_1, \tau_2)^*$ - closed.

Definition 2.2: A map $f: X \to Y$ is called **$(\tau_1, \tau_2)^*$ -Q^* continuous** if the inverse image of each $(\sigma_1, \sigma_2)^*$ - Q^* closed in Y is $\tau_1 \tau_2$ - closed in X.

Definition 2.3 [6]: A map $f: X \to Y$ is called **$\tau_1 \tau_2$ - Q^* - continuous** if the inverse image of each $\sigma_1\sigma_2$ - Q^* closed in Y is τ_2 - closed in X.

Definition2.4: A subset S of X is said to be (τ_1, τ_2) *-semi open set if $S \subseteq \tau_1 \tau_2$ cl $(\tau_1 \tau_2$ - int (S)). The complement of $(\tau_1, \tau_2)^*$ - semi open set is (τ_1, τ_2) * - semi closed.

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3. $(\tau_1, \tau_2)^*$ - $Q^\star$ HOMEOMORPHISM

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2). We denote the family of all **$(\tau_1, \tau_2)^*$ - $Q^\star$** homeomorphisms from (X, τ_1, τ_2) onto itself by **$(\tau_1, \tau_2)^*$ - $Q^\star$** H(X) and the family of all $(\tau_1, \tau_2)^*$ - closed set in (X, τ_1, τ_2) is denoted by $(\tau_1, \tau_2)^*$ - C (X).

Definition 3.1[8]: A bijection f: $X \to Y$ is called **$(\tau_1, \tau_2)^*$ - Q^* homeomorphism**, if f is $(\tau_1, \tau_2)^*$ - Q^* continuous and its inverse also $(\tau_1, \tau_2)^*$ - Q^* continuous.

Example 3.1: Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{b\}, \{b, c\}\}$ and $\sigma_1 = \{\phi, Y, \{b\}\}$, $\sigma_2 = \{\phi, Y, \{b\}, \{b, c\}\}$. Then ϕ, {a}, {a, c} are $(\sigma_1, \sigma_2)^*$- Q^* closed in Y. Let f: $X \to Y$ be the identity map. Then f $(\phi) = \phi$, f ({a, c}) = {a, c}, f({a}) = {a}. Since ϕ, {a, c}, {a} are $\tau_1\tau_2$ - closed in X. Therefore, f and f^{-1} are $(\tau_1, \tau_2)^*$ - Q^* continuous. Hence f is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Definition 3.2: A subset S of X is called **pairwise $(\tau_1, \tau_2)^*$- Q^* homeomorphism** in X if S is both $(\tau_1, \tau_2)^*$ - Q^* homeomorphism and $(\tau_2, \tau_1)^*$ - Q^* homeomorphism.

Definition 3.3: A space (X, τ_1, τ_2) is called **$(\tau_1, \tau_2)^*$- Q^*space** if every $(\tau_1, \tau_2)^*$ - Q^*closed is $(\tau_1, \tau_2)^*$ **-** closed.

Proposition 3.1: If f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and g: $(Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ are $(\tau_1, \tau_2)^*$ - Q^* homeomorphisms, then $g \circ f$: $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is also a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Proof: Let U be a $(\eta_1, \eta_2)^*$ - Q^* open set in (Z, η_1, η_2).

Now $(g\circ f)^{-1}$ (U) = $f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$.

By hypothesis, V is $(\sigma_1, \sigma_2)^*$- Q^* open in (Y, σ_1, σ_2) and again by hypothesis, $f^{-1}(V)$ is $(\tau_1, \tau_2)^*$ -Q^* open in (X, τ_1, τ_2).

Therefore, $(g\circ f)$ is $(\tau_1, \tau_2)^*$ - Q^* continuous.

Also for a $(\tau_1, \tau_2)^*$ - Q^*open set G in (X, τ_1, τ_2),

We have $(g \circ f)$ (G) = g (f (G)) = g (W), where W = f (G).

By hypothesis,
f (G) is $(\sigma_1, \sigma_2)^*$- Q^* open in $(Y\ \sigma_1, \sigma_2)$ and again by hypothesis, g (W) is $(\eta_1, \eta_2)^*$-Q^* open in (Z, η_1, η_2).

Therefore, $(g\circ f)^{-1}$ is $(\tau_1, \tau_2)^*$ - Q^* continuous.

Hence $g\circ f$ is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Example 3.2: Let X = Y = Z = {a, b, c}, $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{b\}, \{b, c\}\}$ and $\sigma_1 = \{\phi, Y, \{b\}\}$, $\sigma_2 = \{\phi, Y, \{b\},\{b, c\}\}$, $\eta_1 = \{\phi, Z, \{b\}, \{b, c\}\}$, $\eta_2 = \{\phi, Z, \{b, c\}\}$. Then ϕ, {a}, {a, c} are $(\sigma_1, \sigma_2)^*$- Q^* closed in Y. Let f: $X \to Y$ be the identity map. Then f and g are $(\tau_1, \tau_2)^*$ - Q^* homeomorphism. Here $g \circ f$ is $(\tau_1, \tau_2)^*$ - Q^* continuous, since {b, c} is $(\eta_1, \eta_2)^*$ - Q^* open in Z and $(g\circ f)^{-1}$ ({b, c}) = {b, c} is $(\tau_1, \tau_2)^*$ - Q^* open in (X, τ_1, τ_2). Hence $g \circ f$ is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Proposition 3.2: The set $(\tau_1, \tau_2)^*$- Q^* H (X) is a group.

Proof: Define Ψ: $(\tau_1, \tau_2)^*$ - Q^* H (X) × $(\tau_1, \tau_2)^*$ - Q^* H (X) → $(\tau_1, \tau_2)^*$ - Q^* (X) by Ψ (f, g) = $(g\circ f)$ for every f, g $\in(\tau_1, \tau_2)^*$ - Q^* H (X).

Then by proposition 3.1, $(g \circ f) \in (\tau_1, \tau_2)^*$ - Q^* (X).

Hence $(\tau_1, \tau_2)^*$ - Q^* H (X) is closed.

We know that the composition of maps is associative.

The identity map
i: $(X, \tau_1,\tau_2) \to (X, \tau_1,\tau_2)$ is a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism and i $\in(\tau_1, \tau_2)^*$-Q^*H (X).

Also $i \circ f = f \circ i = f$ for every $f \in (\tau_1, \tau_2)^*$- Q* H (X).

For any $f \in (\tau_1, \tau_2)^*$ - Q* H (X),

$f \circ f^{-1} = f^{-1} \circ f = i$.

Hence inverse exists for each element of $(\tau_1, \tau_2)^*$- Q* H (X).

Thus, $(\tau_1, \tau_2)^*$- Q* H (X) is a group under composition of maps.

Theorem 3.1: Every $(\tau_1, \tau_2)^*$ - Q^* homeomorphism is a $(\tau_1, \tau_2)^*$ - homeomorphism.

Proof: Let f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Then f is bijective and both f and f^{-1} are $(\tau_1, \tau_2)^*$ - Q^* continuous.

Since every $(\tau_1, \tau_2)^*$- Q* continuous function is $(\tau_1, \tau_2)^*$ continuous we have f and $f^{-1}$are $(\tau_1, \tau_2)^*$ - continuous.

This shows that f is a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Remark 3.1: The converse of the above theorem need not be true, as shown in the following example.

Example 3.3: In example 3.1, f is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism but not $(\tau_1, \tau_2)^*$ - homeomorphism.

Proposition 3.3: If f: $(X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ and g: $(Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$ are $(\tau_1, \tau_2)^*$ - Q^* homeomorphisms, then $g \circ f$: $(X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$ is also a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Proof: Let U be a $\eta_1 \eta_2$ - Q* open set in (Z, η_1, η_2).

Now $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$.

By hypothesis, V is $\sigma_1 \sigma_2$- Q* open in $(Y\ \sigma_1, \sigma_2)$ and again by hypothesis, $f^{-1}(V)$ is $\tau_1 \tau_2$ - Q^*open in (X, τ_1, τ_2).

Therefore, $(g \circ f)$ is $\tau_1 \tau_2$ - Q^* continuous.

Also for a $\tau_1 \tau_2$ - Q^*open set G in (X, τ_1, τ_2),

We have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$.

By hypothesis,

f (G) is $\sigma_1 \sigma_2$- Q* open in $(Y\ \sigma_1, \sigma_2)$ and again by hypothesis, g (W) is $\eta_1 \eta_2$- Q* open in (Z, η_1, η_2).

Therefore, $(g \circ f)^{-1}$ is $\tau_1 \tau_2$ - Q^*continuous.

Hence $g \circ f$ is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Example 3.4: Let X = Y = Z = {a, b, c}, $\tau_1 = \{\phi, X\}$, $\tau_2 = \{\phi, X, \{a\}, \{a, c\}\}$ and $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{a, c\}\}$, $\eta_1 = \{\phi, Z, \{a\}, \{a, c\}\}$, $\eta_2 = \{\phi, Z, \{a, c\}\}$. Then ϕ, {b, c}, {b} are $\sigma_1 \sigma_2$ - Q^* closed in Y. Let f: $X \rightarrow Y$ be the identity map. Then f and g are $(\tau_1, \tau_2)^*$ - Q^* homeomorphism .Here $g \circ f$ is $\tau_1 \tau_2$- Q* continuous, since {a, c} is $\eta_1 \eta_2$ - Q* open in Z and $(g \circ f)^{-1}(\{a, c\}) = \{a, c\}$ is $\tau_1 \tau_2$- Q* open in (X, τ_1, τ_2). Hence $g \circ f$ is $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Proposition 3.4: The set $\tau_1 \tau_2$ - Q^*H(X, τ_1, τ_2) is a group.

Proof: Define Ψ: $\tau_1 \tau_2$ - Q^*H $(X, \tau_1, \tau_2) \times \tau_1 \tau_2$ - Q^*H $(X, \tau_1, \tau_2) \rightarrow \tau_1 \tau_2$ - Q^*H(X, τ_1, τ_2) by $\Psi(f, g) = (g \circ f)$ for every f, g $\in \tau_1 \tau_2$ - Q^*H (X, τ_1, τ_2).

Then by proposition 3.3, $(g \circ f) \in \tau_1 \tau_2$ - Q^*H (X, τ_1, τ_2).

Hence $\tau_1 \tau_2$ - Q^*H(X, τ_1, τ_2) is closed.

We know that the composition of maps is associative.

The identity map
i: $(X, \tau_1, \tau_2) \to (X, \tau_1, \tau_2)$ is a $\tau_1 \tau_2$ - Q★ homeomorphism and i $\in \tau_1 \tau_2$ - $Q^*H(X, \tau_1, \tau_2)$.

Also $i \circ f = f \circ i = f$ for every f $\in \tau_1 \tau_2$ - $Q^*H(X, \tau_1, \tau_2)$.

For any f $\in \tau_1 \tau_2$ - Q^*H (X, τ_1, τ_2),

$f \circ f^{-1} = f^{-1} \circ f = i$.

Hence inverse exists for each element of $\tau_1 \tau_2$ - $Q^*H(X, \tau_1, \tau_2)$.

Thus, $\tau_1 \tau_2$ - $Q^*H(X, \tau_1, \tau_2)$ is a group under composition of maps.

Theorem3.2 -Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map. Then the following are true.

i) Every $\tau_1\tau_2$- Q★ homeomorphism is $\tau_1\tau_2$- homeomorphism.

ii) Every $(\tau_1, \tau_2)^*$ - Q^* homeomorphism is $\tau_1\tau_2$ - Q★ homeomorphism.

Proof: The proof is obvious.

Definition 3.4: For a subset A of a space (X, τ_1, τ_2) we define the **$(\tau_1, \tau_2)^*$ - Q* kernel** of A (briefly, $(\tau_1, \tau_2)^*$ - Q* ker(A)) as follows: $(\tau_1, \tau_2)^*$ - Q* ker(A) = $\cap$ {F: F $\in(\tau_1, \tau_2)^*$ -Q* O (X, τ_1, τ_2); A $\subset$ F}. A is said to be a **$(\tau_1, \tau_2)^*$ - Q* - Λset** in (X, τ_1,τ_2) if A = $(\tau_1, \tau_2)^*$ - Q* ker(A), or equivalently, if A is the intersection of $(\tau_1, \tau_2)^*$ - Q* open sets. A is said to be **$(\tau_1, \tau_2)^*$ - Q* λ - closed** in (X, τ_1, τ_2) if it is the intersection of a $(\tau_1, \tau_2)^*$ - Q* - Λ set in (X, τ_1,τ_2) and a quasi closed set in (X, τ_1, τ_2) Clearly, $(\tau_1, \tau_2)^*$ - Q* - Λ set sand $(\tau_1, \tau_2)^*$ - Q* closed sets are $(\tau_1, \tau_2)^*$ - Q* - λclosed; complements of $(\tau_1, \tau_2)^*$ - Q* - λ closed sets in (X, τ_1 ,τ_2) are said to be $(\tau_1, \tau_2)^*$ - Q* - λopen in (X, τ_1, τ_2).

Proposition 3.5: For a subset A of a space (X, τ_1, τ_2), the following are equivalent:
(i) A is $(\tau_1, \tau_2)^*$ - Q* - λ closed in (X, τ_1, τ_2).
(ii) A = L $\cap(\tau_1, \tau_2)^*$ - Q* cl(A), where L is a $(\tau_1, \tau_2)^*$ - Q* - Λ set in (X, τ_1, τ_2).
(iii) A = $(\tau_1, \tau_2)^*$ - Q* ker(A) $\cap(\tau_1, \tau_2)^*$ - Q* cl(A).

Definition 3.5: A bijection f: X $\to$ Y is called $(\tau_1, \tau_2)^*$ - Q^* **homeomorphism**, if f is $(\tau_1, \tau_2)^*$ - Q^* irresolute and its inverse also $(\tau_1, \tau_2)^*$ - Q^* irresolute.

Remark 3.2: We say that spaces (X, τ_1, τ_2) and $(Y \sigma_1, \sigma_2)$ are $(\tau_1, \tau_2)^*$ - Q^* homeomorphic if there exists a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism from (X, τ_1, τ_2) onto (Y, σ_1, σ_2).

Theorem 3.3: Every $(\tau_1, \tau_2)^*$ - Q★ homeomorphism is a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Proof: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Then f is bijective, $(\tau_1, \tau_2)^*$ - Q^* irresolute and f^{-1} is $(\tau_1, \tau_2)^*$ - Q^* irresolute.

Since every $(\tau_1, \tau_2)^*$ - Q^* irresolute is $(\tau_1, \tau_2)^*$ - Q^* continuous, f and f^{-1} are $(\tau_1, \tau_2)^*$ - Q^* continuous and so f is a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Remark 3.3: The following example shows that the converse of the above theorem need not be true.

Example 3.5: Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}, {c},{a, c}} and τ_2 = {ϕ, X,{a}}. Clearly {b} is $(\tau_1, \tau_2)^*$ - Q^* closed in X .Let σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y}.

Then $\sigma_1\sigma_2$ - open sets on Y are ϕ, Y,{a} and $\sigma_1\sigma_2$ - closed sets on X are ϕ, Y, {b, c}. Since {b, c} is $(\sigma_1, \sigma_2)^*$ - Q* closed in Y but f^{-1}({b, c}) = {b, c} is not $(\tau_1, \tau_2)^*$ - Q^* open in X and so f is not $(\tau_1, \tau_2)^*$ - Q* irresolute.

Remark 3.4: The above example shows that the concepts of $(\tau_1, \tau_2)^*$-homeomorphisms and $(\tau_1, \tau_2)^*$-Q^* homeomorphism are independent.

Definition 3.6 -A map f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be **$(\tau_1, \tau_2)^*$ - Q^*closed** if for every $(\tau_1, \tau_2)^*$ - Q* closed F of X, f (F) is $(\tau_1, \tau_2)^*$ - closed in Y.

Proposition 3.6 - For any bijection f: $X \to Y$, the following statements are equivalent.
(a) f^{-1}: $Y \to X$ is $(\tau_1, \tau_2)^*$ - Q^* continuous.
(b) f is a $(\tau_1, \tau_2)^*$ - Q^* open map.
(c) f is a $(\tau_1, \tau_2)^*$ - Q^* closed map.

Proof:
Step 1: (a) ⇒ (b)
Let V be a $(\tau_1, \tau_2)^*$ - open set in X.

Then X – V is $(\tau_1, \tau_2)^*$ - closed in X.

Since f^{-1} is $(\tau_1, \tau_2)^*$ - Q* continuous,

$(f^{-1})^{-1}(X – V) = f (X – V) = Y – f (V)$ is $(\tau_1, \tau_2)^*$ - Q* closed in Y.

Then f (V) is $(\tau_1, \tau_2)^*$ - Q* open in Y.

Hence f is a $(\tau_1, \tau_2)^*$ - Q* open map.

Step 2: (b) ⇒(c).
Let f be a $(\tau_1, \tau_2)^*$ - Q* open map.

Let U be a$(\tau_1, \tau_2)^*$ - closed set in X.

Then X – U is $(\tau_1, \tau_2)^*$ - open in X.

Since f is $(\tau_1, \tau_2)^*$ - Q* open,

f(X – U)=Y – f(U) is $(\tau_1, \tau_2)^*$ - Q* open in Y.

Then f (U) is $(\tau_1, \tau_2)^*$ - Q* closed in Y.

Hence f is a $(\tau_1, \tau_2)^*$ - Q* closed.

Step 3: (c) ⇒ (a)**.**
Let V be $(\tau_1, \tau_2)^*$ - Q* closed set in X.

Since f: X→Y is $(\tau_1, \tau_2)^*$ - Q* closed,

f(V) is $(\tau_1, \tau_2)^*$ - Q* closed in Y.

That is $(f^{-1})^{-1}(V)$ is $(\tau_1, \tau_2)^*$ - Q* closed in Y.

Hence f^{-1} is $(\tau_1, \tau_2)^*$ - Q* continuous.

Proposition 3.7: Let f: ~~X~~→Y be a bijective and $(\tau_1, \tau_2)^*$ - Q* continuous map. Then the following statements are equivalent.
(a) f is a $(\tau_1, \tau_2)^*$ - Q* open map.
(b) f is a $(\tau_1, \tau_2)^*$ - $Q^\star$ homeomorphism.
(c) f is a $(\tau_1, \tau_2)^*$ - Q* closed map.

Proof:
Step 1: (a) ⇒ (b).
Given f is bijective, $(\tau_1, \tau_2)^*$ - Q* continuous map and $(\tau_1, \tau_2)^*$ - Q* open map. Hence f is $(\tau_1, \tau_2)^*$-Q^* homeomorphism.

Step 2: (b) ⇒(c).
Let f be a $(\tau_1, \tau_2)^*$ - Q^* homeomorphism.

Hence f is $(\tau_1, \tau_2)^*$ - Q* open.

By Proposition 3.6, f is $(\tau_1, \tau_2)^*$ - Q* closed.

Step 3: (c) ⇒ (a)

Follows from Proposition 3.6.

Definition 3.7: Let S be a subset of X. Let $x \in X$. Then x is said to be a $(\tau_1, \tau_2)^*$ - Q* limit point of S if and only if every $(\tau_1, \tau_2)^*$ - Q* open set containing x contains at least one point other than x.

Definition 3.8: Let S be a subset of X. Then the set of all $(\tau_1, \tau_2)^*$ - Q* limit points of S is said to be $(\tau_1, \tau_2)^*$ - Q* derived set of S and it is denoted by $(\tau_1, \tau_2)^*$ - D Q* (S).

Theorem 3.4: Let A be a subset of X. Let $(\tau_1, \tau_2)^*$ - D Q* (A) be the set of all $(\tau_1, \tau_2)^*$ - Q* limit points of A. Then $(\tau_1, \tau_2)^*$ - Q* cl(A) = A $\cup(\tau_1, \tau_2)^*$ - D Q* (A).

Proof: Let $x \in A \cup (\tau_1, \tau_2)^*$ - D Q* (A).

This implies either $x \in A$ or $x \in(\tau_1, \tau_2)^*$ - D Q* (A).

If $x \in A$, then $x \in (\tau_1, \tau_2)^*$ - Q* cl(A).

If $x \in(\tau_1, \tau_2)^*$ - D Q* (A), then every $(\tau_1, \tau_2)^*$ - Q* open set contains x will intersect with A.

Therefore, $x \in (\tau_1, \tau_2)^*$ - Q* cl(A).

This implies $A \cup (\tau_1, \tau_2)^*$ - D Q* (A) $\subseteq (\tau_1, \tau_2)^*$ - Q* cl(A).

If $x \in(\tau_1, \tau_2)^*$ - Q* cl(A), then to prove $x \in A \cup (\tau_1, \tau_2)^*$ - D Q* (A).

If $x \in A$, then $x \in A \cup (\tau_1, \tau_2)^*$ - D Q* (A).

If $x \notin A$, since $x \in (\tau_1 , \tau_2)^*$ - Q* cl(A) implies every $(\tau_1, \tau_2)^*$ - Q* open set of x intersects with A.

Hence $x \in (\tau_1, \tau_2)^*$ - D Q* (A).

Therefore, $(\tau_1, \tau_2)^*$ - Q* cl(A) = A $\cup (\tau_1, \tau_2)^*$ - D Q* (A).

Definition 3.9: Let S be a subset of X. Any point of $(\tau_1, \tau_2)^*$ - Q* cl (S) is referred to as a $(\tau_1, \tau_2)^*$ - Q* contact (or adherent) point of S.

Definition 3.10 [6]: A bijection f: X → Y is called **$\tau_1 \tau_2$ - Q★ homeomorphism**, if f is $\tau_1 \tau_2$ - Q^* continuous and its inverse also $\tau_1 \tau_2$ - Q^* - continuous.

Example 3.6: In example 3.1, ϕ, {a}, {a, c} are $\sigma_1 \sigma_2$ - Q^* closed in Y. Let f: X → Y be the identity map. Then f (ϕ) = ϕ, f ({a, c}) = {a, c}, f ({a}) = {a}. Since ϕ, {a, c}, {a} are τ_2 - closed in X. Therefore, f and f^{-1} are $\tau_1 \tau_2$ - Q^* continuous. Hence f is $\tau_1\tau_2$ - Q^* homeomorphism.

Definition 3.11: A bijection f: X → Y is called $\tau_1 \tau_2$ - Q★ **homeomorphism**, if f is $\tau_1\tau_2$ - Q^* irresolute and its inverse also $\tau_1\tau_2$ - Q^* irresolute.

Theorem 3.3: Every $\tau_1\tau_2$ - Q★ homeomorphism is a $\tau_1 \tau_2$ - Q★ homeomorphism.

Proof: Let f: $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a $\tau_1 \tau_2$ - Q★ homeomorphism.

Then f is bijective, $\tau_1\tau_2$ - Q^* irresolute and f^{-1} is $\tau_1\tau_2$ -Q^* irresolute.

Since every $\tau_1\tau_2$ - Q^* irresolute is $\tau_1\tau_2$ - Q^* continuous, f and f^{-1} are $\tau_1\tau_2$ - Q^* continuous and so f is a $\tau_1\tau_2$ - Q^* homeomorphism.

Remark 3.3: The following example shows that the converse of the above theorem need not be true.

Example 3.7: Let X = Y = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, c}} and τ_2 = {ϕ, X,{a}}. Clearly {b, c} is $\tau_1\tau_2$ - Q^* closed in X. Let σ_1 = {ϕ, Y, {a}} and σ_2 = {ϕ, Y}. Then $\sigma_1\sigma_2$ - open sets on Y are ϕ, Y, {a} and $\sigma_1\sigma_2$ - closed sets on X are ϕ, Y, {b, c}. Since {b, c} is $\sigma_1\sigma_2$ - Q* closed in Y but f^{-1}({b, c}) = {b, c} is not$\tau_1\tau_2$ - Q^* open in X and so f is not $\tau_1\tau_2$ - Q* irresolute.

Remark 3.4: The above example shows that the concepts of $\tau_1\tau_2$ - homeomorphisms and $\tau_1\tau_2$ - Q★ homeomorphism are independent.

Definition 3.12: A bitopological space (X, τ_1, τ_2) is called (τ_1, τ_2)* - irreducible if X is not empty and whenever X = $A_1 \cup A_2$ with (τ_1, τ_2)* - closed subsets $A_i \in$ X (i = 1, 2) then we have X = A_1 or A_2.

Example: Let X = {1, 2, 3} and τ_1 = {ϕ, X, {1}, {1, 2}} and τ_2 = {ϕ, X, {1}}. Then (τ_1, τ_2)* - closed sets are ϕ, X,{2, 3}, {3}. Then X is (τ_1, τ_2)* - irreducible.

Theorem: A bitopological space X is (τ_1, τ_2)* -irreducible if and only if every nonempty open set is (τ_1, τ_2)* - Q* open.

Proof: Let X be a (τ_1, τ_2)* - irreducible.

Let U be any nonempty open set.

If U = X then nothing to prove.

Let U $\neq$ X.

Then (τ_1, τ_2)* - cl (U) $\neq$ X.

Then there exists an (τ_1, τ_2)* - open set V such that U $\cap$ V =ϕ.

This implies $U^c \cap V^c$ = X, where $U^c$and V^c are proper (τ_1, τ_2)* - closed sets which is a contradiction to the fact that X is (τ_1, τ_2)* - irreducible.

Conversely assume that every (τ_1, τ_2)* - open set is (τ_1, τ_2)* - Q* open.

We claim that X is (τ_1, τ_2)* - irreducible.

Then X = A $\cup$ B, where A and B are proper nonempty (τ_1, τ_2)* - closed sets.

$A^c \cap B^c$ = ϕ.

Then $A^c$is not dense.

Then A^c is an (τ_1, τ_2)* - open set but not (τ_1, τ_2)* - Q* open.

Hence X is irreducible.

Definition 3.13: A bitopological space (X, τ_1, τ_2) is called $\tau_i\tau_j$ - irreducible if X is not empty and whenever X = $A_1 \cup A_2$ with τ_i- closed subset $A_1 \in$ X and τ_j- closed subset $A_2 \in$ X (i, j = 1, 2) then we have X = A_1 or A_2.

Example: Let X = {1, 2, 3} and τ_1 = {ϕ, X, {1}, {1, 3}} and τ_2 = {ϕ, X, {2},{2, 3}} . Then τ_1- closed sets are ϕ, X, { 2, 3}, {2} and τ_2- closed sets are ϕ, X, {1, 3}, {1}. Then X is $\tau_i\tau_j$-irreducible.

Theorem: A bitopological space X is $\tau_i\tau_j$- irreducible if and only if every nonempty open set is $\tau_i\tau_j$- Q* open.

Proof: Let X be a $\tau_i\tau_j$- irreducible.

Let U be any nonempty τ_j- open set.

If U = X then nothing to prove.

Let U $\neq$ X.

Then τ_i - cl (U) $\neq$ X.

Then there exists an τ_i- open set V such that U $\cap$ V =ϕ.

This implies $U^c \cap V^c$ = X, where U^c and V^c are proper τ_i - closed set and τ_j-closed set which is a contradiction to the fact that X is $\tau_i\,\tau_j$ - irreducible.

Conversely assume that every $\tau_i\,\tau_j$ - open set is $\tau_i\,\tau_j$ - Q* open.

We claim that X is $\tau_i\,\tau_j$- irreducible.

Then X = A $\cup$B, where A and B are proper nonempty τ_i - closed set and τ_j - closed set.

$A^c \cap B^c$ = ϕ.

Then A^c is not dense.

Then A^c is an $\tau_i\,\tau_j$ - open set but not $\tau_i\,\tau_j$ - Q* open.

Hence X is irreducible.

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