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A NEW GENERALISATION OF SAM-SOLAI'S MULTIVARIATE ADDITIVE F- DISTRIBUTION OF KIND-2*

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ABSTRACT

This paper proposed a new generalization of Sam-Solai's Multivariate additive F- distribution of Kind-2 from the uni-variate case. Further, we find its Marginal, Multivariate Conditional distributions, Multivariate Generating functions, Multivariate survival, hazard functions and also discussed its special cases. The authors derived the generating functions of this distribution in terms of Whittaker-M function and complex Meijer-G function. The special cases includes the transformation of Sam-Solai's Multivariate additive F- distribution of Kind -2 into Multivariate additive Beta distribution of Kind-1 of Type-B, Multivariate Beta-distribution of Kind-2 of Type-B, Multivariate Fisher's Z-distribution of Kind-2 and Multivariate Logistic-F distribution of kind-2. Moreover, it is found that the bi-variate correlation between two F- variables purely depends on the d.f and we simulated and established selected standard bi-variate F- correlation bounds from 10,000 different combination of d.f. The simulation results shows, the correlation between any two F- variables bounded from -1 to +1 for certain combination of fractional d.f.

Keywords: *Sam-Solai's Multivariate additive F- distribution of Kind-2, Transformation, Multivariate additive Beta distribution of Kind-1 of Type-B, Multivariate Beta-distribution of Kind-2 of Type-B, Multivariate Fisher's Z-distribution of Kind-2, Multivariate Logistic-F distribution of kind-2 ,Correlation bounds, Fractional d.f*

Mathematics Subject Classification: *Primary 62H10; Secondary 62E15.*

INTRODUCTION

The pioneering work of R.A. Fisher and George W. Snedecor gave a birth of F-distribution to the statistical literature and the origin of F- distribution, its multivariate generalization was extensively studied because of the wide applications of the distribution in various fields for the past five decades. Many authors attempted to give an alternate form of multivariate generalization of F-distribution. At first, Leo Aroian [1941] studied the F-distribution and its relationship with Fisher's Z distribution and Seymour Geisser *et al.* [1958] analyzed the results proposed by Box and extend the results to study the application of F-distribution in Multivariate analysis. Similarly Nico Laubscher [1960] proposed the procedure of normalizing the Non-central t and F-distributions and Tiao *et al.* [1965] emphasized that the F-distribution is a special case of inverted Dirichlet distribution. Moreover, Konno [1988], Fang *et al.* [1990], Yasunori Fujikoshi [1993] studied the exact moments, symmetricity, Quantile approximation with special reference to the F-distribution respectively. On the other hand, Leung [1994] found the identity for the non-central wishart distribution which includes the F-distribution and Jones [2001] explored the association among the Multivariate Beta, t and F-distributions. Likewise, Pham Gia *et al.* [2003] and Arnold *et al.* [2006] studied the application of F-distribution in statistical modeling and the Rosenblatt construction of multivariate family of distributions which includes F-distribution respectively.Finally,El-Bassiouny et al [2007] proposed the bivariate –F distribution with special marginals and Nadarajah [2007] explored the Compound F-distribution and discussed its properties. From the in-depth reviews of F- distribution, this paper proposed an alternate form of multivariate F- distribution and its structure, form were discussed in the next section.

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SECTION-1: SAM-SOLAI'S MULTIVARIATE ADDITIVE F- DISTRIBUTION

Definition 1.1: Let $F_1, F_2, F_3 \ldots, F_p$ are the random variables followed Continuous uni-variate F- distribution with (m_i, n_i) degrees of freedom for all i (*i=1 to p*), then the Multivariate Sam-Solai's additive F- distribution of Kind-2 and its density function is defined as

$$g(F_1, F_2, F_3 \ldots, F_p) = \left\{ \left(\sum_{i=1}^{p} \frac{B\left(\frac{m_i}{2}, 1\right)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, 1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}} \tag{1}$$

where $0 \le F_i \le \infty$, $m_i, n_i > 0$

Theorem 1.2: The cumulative distribution function of the Sam-Solai's Multivariate additive F- distribution is defined by

$$G(F_1, F_2, F_3 \ldots, F_p) = \int_0^{F_1}\int_0^{F_2}\int_0^{F_3} \ldots \int_0^{F_p} \left\{ \left(\sum_{i=1}^{p} \frac{B(\frac{m_i}{2}, 1)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1+(m_i/n_i)u_i)^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, 1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}} du_i \tag{2}$$

where, $0 \le u_i < F_i$, $m_i, n_i > 0$

$$G(F_1, F_2, F_3 \ldots, F_p) = \left\{ \left(\sum_{i=1}^{p} \frac{\int_0^{F_i} \frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+\frac{n_i}{2}}} du_i}{\int_0^{F_i} \frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, 1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}} du_i} \right) - (p-1) \right\} \prod_{i=1}^{p} \int_0^{F_i} \frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, 1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}} du_i$$

$$G(F_1, F_2, F_3 \ldots, F_p) = \left\{ \left(\sum_{i=1}^{p} \frac{\phi_i(F_i; m_i, n_i)}{\omega_i(F_i; m_i, n_i)} \right) - (p-1) \right\} \prod_{i=1}^{p} \omega_i(F_i; m_i, n_i)$$

where $\phi_i(F_i; m_i, n_i) = \int_0^{F_i} \frac{(m_i/n_i)^{\frac{m_i}{2}} u_i^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+\frac{n_i}{2}}} du_i$ and $\omega_i(F_i; m_i, n_i) = \int_0^{F_i} \frac{(m_i/n_i)^{\frac{m_i}{2}} u_i^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2}, 1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}} du_i$ are the lower incomplete Beta integrals of i^{th} F- variable.

Theorem 1.3: The Probability density function of Sam-Solai's Multivariate additive Conditional F- distribution of F_1 on $F_2, F_3 \ldots, F_p$ is

$$g(F_1 / F_2, F_3 \ldots, F_p) = \frac{\left\{ \left(\sum_{i=1}^{p} \frac{B(\frac{m_i}{2}, 1)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} \frac{(m_1/n_1)((m_1/n_1)F_1)^{\frac{m_1}{2}-1}}{B\left(\frac{m_1}{2}, 1\right)(1+(m_1/n_1)F_1)^{\frac{m_1}{2}+1}}}{\left\{ \left(\sum_{i=2}^{p} \frac{B(\frac{m_i}{2}, 1)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}} \right) - (p-2) \right\}} \tag{3}$$

where $0 \le F_1 \le \infty$, $m_i, n_i > 0$

Proof: It is obtained from

$$g(F_1 / F_2, F_3 \ldots, F_p) = \frac{g(F_1, F_2, F_3 \ldots, F_p)}{g(F_2, F_3 \ldots, F_p)}$$

Theorem 1.4: Mean and Variance of Sam - Solai's Multivariate additive Conditional F- distribution are

$$E(F_1/F_2,F_3\ldots,F_p)=\frac{\dfrac{1}{(m_1/n_1)}\left\{\dfrac{B\left(\frac{m_1}{2}+1,\frac{n_1}{2}-1\right)}{B\left(\frac{m_1}{2},\frac{n_1}{2}\right)}+\dfrac{1}{B\left(\frac{m_1}{2},1\right)}\left(\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right)\right\}}{\left\{\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}} \tag{4}$$

$$V(F_1/F_2,F_3\ldots,F_p)=E(F_1^2/F_2,F_3\ldots,F_p)-(E(F_1/F_2,F_3\ldots,F_p))^2 \tag{5}$$

where

$$E(F_1^2/F_2,F_3\ldots,F_p)=\frac{\dfrac{1}{(m_1/n_1)^k}\left\{\dfrac{B\left(\frac{m_1}{2}+2,\frac{n_1}{2}-2\right)}{B\left(\frac{m_1}{2},\frac{n_1}{2}\right)}+\dfrac{B\left(\frac{m_1}{2}+2,-1\right)}{B\left(\frac{m_1}{2},1\right)}\left(\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right)\right\}}{\left\{\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}}$$

Proof: The k^{th} order moment of the distribution is

$$E(F_1^k/F_2,F_3\ldots,F_p)=\int_0^\infty F_1^k g(F_1/F_2,F_3\ldots,F_p)dF_1$$

$$E(F_1^k/F_2,F_3\ldots,F_p)=\int_0^\infty F_1^k\frac{\left\{\left(\sum_{i=1}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right\}\dfrac{(m_1/n_1)((m_1/n_1)F_1)^{\frac{m_1}{2}-1}}{B\left(\frac{m_1}{2},1\right)(1+(m_1/n_1)F_1)^{\frac{m_1}{2}+1}}}{\left\{\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}}dF_1$$

$$E(F_1^k/F_2,F_3\ldots,F_p)=\frac{\dfrac{1}{(m_1/n_1)^k}\left\{\dfrac{B\left(\frac{m_1}{2}+k,\frac{n_1}{2}-k\right)}{B\left(\frac{m_1}{2},\frac{n_1}{2}\right)}+\dfrac{B\left(\frac{m_1}{2}+k,1-k\right)}{B\left(\frac{m_1}{2},1\right)}\left(\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right)\right\}}{\left\{\left(\sum_{i=2}^{p}\dfrac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}}$$

If ***k=1***, then the Conditional expectation is

$$E(F_1/F_2,F_3\ldots,F_p)=\frac{\dfrac{1}{(m_1/n_1)}\left\{\dfrac{B\left(\frac{m_1}{2}+1,\frac{n_1}{2}-1\right)}{B\left(\frac{m_1}{2},\frac{n_1}{2}\right)}+\dfrac{1}{B(\frac{m_1}{2},1)}\left(\left(\sum_{i=2}^{p}\dfrac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right)\right\}}{\left\{\left(\sum_{i=2}^{p}\dfrac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\dfrac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}}$$

If ***k=2***, then the second order moment is

$$E(F_1^2/F_2,F_3\dots,F_p)=\frac{\frac{1}{(m_1/n_1)^k}\left\{\frac{B\left(\frac{m_1}{2}+2,\frac{n_1}{2}-2\right)}{B\left(\frac{m_1}{2},\frac{n_1}{2}\right)}+\frac{B\left(\frac{m_1}{2}+2,-1\right)}{B\left(\frac{m_1}{2},1\right)}\left(\left(\sum_{i=2}^{p}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right)\right\}}{\left\{\left(\sum_{i=2}^{p}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-2)\right\}}$$

The conditional variance of the distribution is obtained by Substituting the first and second moments in (5).

Theorem 1.5: If there are ***p = (q + k)*** random variables, such that ***q*** random variables $F_1,F_2,F_3,\dots F_q$ conditionally depends on the ***k*** variables $F_{q+1},F_{q+2},F_{q+3},\dots F_{q+k}$,then the density function of Sam-Solai's multivariate additive conditional F- distribution is

$$g(F_1,F_2,F_3\dots,F_q/F_{q+1},F_{q+2},F_{q+3}\dots,F_{q+k})=\frac{\left\{\left(\sum_{i=1}^{q+k}\frac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(q+k-1)\right\}\prod_{i=1}^{q}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}}{\left\{\left(\sum_{i=q+1}^{q+k}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(k-1)\right\}} \quad (6)$$

where $0\le F_i\le\infty$, $m_i,n_i>0$

Proof: Let the multivariate conditional law for ***q*** random variables $F_1,F_2,F_3,\dots F_q$ conditionally depending on the ***k*** variables $F_{q+1},F_{q+2},F_{q+3},\dots F_{q+k}$ is given as

$$g(F_1,F_2,F_3,\dots F_q/F_{q+1},F_{q+2},F_{q+3},\dots F_{q+k})=\frac{g(F_1,F_2,F_3,\dots F_q,F_{q+1},F_{q+2},F_{q+3},\dots F_{q+k})}{g(F_{q+1},F_{q+2},F_{q+3},\dots F_{q+k})}$$

$$g(F_1,F_2,F_3\dots,F_q/F_{q+1},F_{q+2},F_{q+3}\dots,F_{q+k})=\frac{\left\{\left(\sum_{i=1}^{q+k}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(q+k-1)\right\}\prod_{i=1}^{q+k}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}}{\int_0^\infty\int_0^\infty\dots\int_0^\infty\left\{\left(\sum_{i=1}^{q+k}\frac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(q+k-1)\right\}\prod_{i=1}^{q+k}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}\prod_{i=1}^{q}dF_i}$$

$$g(F_1,F_2,F_3\dots,F_q/F_{q+1},F_{q+2},F_{q+3}\dots,F_{q+k})=\frac{\left\{\left(\sum_{i=1}^{q+k}\frac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(q+k-1)\right\}\prod_{i=1}^{q}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}}{\left\{\left(\sum_{i=q+1}^{q+k}\frac{B(\frac{m_i}{2},1)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(k-1)\right\}}$$

where $0\le F_i\le\infty$, $m_i,n_i>0$

SECTION 2: CONSTANTS OF SAM-SOLAI'S MULTIVARIATE ADDITIVE F- DISTRIBUTION

Theorem 2.1: The Marginal product moments, Co-variance and Correlation Co-efficientbetween the random variables F_1 and F_2 are given as

$$E(F_1F_2) = \frac{n_1n_2}{2}\left(\frac{1}{n_1-2}+\frac{1}{n_2-2}-\frac{1}{2}\right) \tag{7}$$

$$COV(F_1,F_2) = \frac{n_1n_2}{2}\left(\frac{n_1+n_2-6}{(n_1-2)(n_2-2)}-\frac{1}{2}\right) \tag{8}$$

$$\rho(F_1,F_2) = \frac{2n_1+2n_2-(n_1-2)(n_2-2)-12}{8\sqrt{\frac{(m_1+n_1-2)(m_2+n_2-2)}{m_1m_2(n_1-4)(n_2-4)}}} \tag{9}$$

where $-1 \le \rho(F_1,F_2) \le +1$ for certain d.f (see Result3.4)

Proof: Assume that F_1 and F_2 are random variables from Sam-Solai's multivariate additive F- distribution. Let the product moment of the distribution is

$$E(F_1F_2) = \int_0^\infty\int_0^\infty \dots \int_0^\infty F_1F_2\, g(F_1,F_2,F_3,\dots F_p)\prod_{i=1}^{p} dF_i$$

Its Co-variance is

$$COV(F_1,F_2) = E(F_1F_2) - E(F_1)E(F_2) \tag{10}$$

Then

$$E(F_1F_2) = \int_0^\infty\int_0^\infty \dots \int_0^\infty F_1F_2\left\{\left(\sum_{i=1}^{p}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right\}\prod_{i=1}^{p}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}\prod_{i=1}^{p}dF_i$$

By evaluation, it follows that $E(F_1F_2) = \frac{n_1n_2}{2}\left(\frac{1}{n_1-2}+\frac{1}{n_2-2}-\frac{1}{2}\right)$.The marginal expectation of the random variables F_1 and F_2 are $E(F_1) = n_1/(n_1-2)$ and $E(F_2) = n_2/(n_2-2)$ respectively. The marginal Product moment for $E(F_1F_2)$ is obtained by substituting the above marginal expectations for F_1 and F_2 in (10).

Thus $COV(F_1,F_2) = \frac{n_1n_2}{2}\left(\frac{n_1+n_2-6}{(n_1-2)(n_2-2)}-\frac{1}{2}\right)$ (11)

Correlation coefficient of a distribution is $\rho(F_1,F_2) = COV(F_1,F_2)/\sigma_1\sigma_2$ (12a)

It observes that $\sigma_1 = (n_1/(n_1-2))\sqrt{2(m_1+n_1-2)/m_1(n_1-4)}$ and $\sigma_2 = (n_2/(n_2-2))\sqrt{2(m_2+n_2-2)/m_2(n_2-4)}$ (12b)

From (11), (12a) and (12b), it follows that

$$\rho(F_1,F_2) = \frac{2n_1+2n_2-(n_1-2)(n_2-2)-12}{8\sqrt{\frac{(m_1+n_1-2)(m_2+n_2-2)}{m_1m_2(n_1-4)(n_2-4)}}} \tag{13}$$

where $-1 \le \rho(F_1,F_2) \le +1$ for certain d.f

Remark 2.1: The Product moments, Co-variance and Correlation Co-efficient between the $i^{th}$and j^{th} F- variables are given as

$$E(F_iF_j)=\frac{n_in_j}{2}\left(\frac{1}{n_i-2}+\frac{1}{n_j-2}-\frac{1}{2}\right) \tag{14}$$

$$COV(F_i,F_j)=\frac{n_in_j}{2}\left(\frac{n_i+n_j-6}{(n_i-2)(n_j-2)}-\frac{1}{2}\right) \tag{15}$$

$$\rho(F_i,F_j)=\frac{2n_i+2n_j-(n_i-2)(n_j-2)-12}{8\sqrt{\dfrac{(m_i+n_i-2)(m_j+n_j-2)}{m_im_j(n_i-4)(n_j-4)}}} \tag{16}$$

where, $i\neq j$, $-1\leq\rho(F_1,F_2)\leq+1$ for certain d.f

Theorem 2.2: The Moment generating function of Sam-Solai's Multivariate additive F- distribution is

$$M_{x_1,x_2\ldots x_P}(t_1,t_2\ldots t_P)=\prod_{i=1}^{p}\varphi(t_i;m_i,n_i)\left(\sum_{i=1}^{p}\frac{\phi(t_i;m_i,n_i)}{\varphi(t_i;m_i,n_i)}-(p-1)\right) \tag{17}$$

Proof: Let the moment generating function of a Multivariate distribution is given as

$$M_{F_1,F_2,F_{3,\ldots}F_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{\sum_{i=1}^{p}t_iF_i}f(F_1,F_2,F_3,\ldots F_p)\prod_{i=1}^{p}dF_i$$

$$M_{F_1,F_2,F_{3,\ldots}F_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{\sum_{i=1}^{p}t_iF_i}\left\{\left(\sum_{i=1}^{p}\frac{B\left(\frac{m_i}{2},1\right)}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right\}\prod_{i=1}^{p}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}\prod_{i=1}^{p}dF_i \tag{17a}$$

From (17a), it observed

$$\frac{1}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)}\int_0^\infty e^{t_iF_i}\frac{(m_i/n_i)^{m_i/2}F_i^{\frac{m_i}{2}-1}}{(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+\frac{n_i}{2}}}dF_i=\frac{(-t_in_i/m_i)^{-m_i/2}e^{-t_in_i/2m_i}}{t_in_i\Gamma(m_i/2)\Gamma(n_i/2)(n_i^2-4)}\sum_{r=1}^{4}\gamma_r(t_i;m_i,n_i)$$

$$\frac{1}{B\left(\frac{m_i}{2},1\right)}\int_0^\infty e^{t_iF_i}\frac{(m_i/n_i)^{m_i/2}F_i^{\frac{m_i}{2}-1}}{(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}dF_i=-\frac{t_in_i(-t_in_i/m_i)^{-\frac{m_i}{2}}}{m_i\Gamma(m_i/2)}G\left([[0],[]],\left[\left[\frac{m_i-1}{2},\frac{m_i}{2}\right],[]\right],\frac{-t_in_i}{m_i}\right)$$

Thus, by integration $M_{x_1,x_2\ldots x_P}(t_1,t_2\ldots t_P)=\prod_{i=1}^{p}\varphi(t_i;m_i,n_i)\left(\sum_{i=1}^{p}\frac{\phi(t_i;m_i,n_i)}{\varphi(t_i;m_i,n_i)}-(p-1)\right)$

where $\phi(t_i;m_i,n_i)=\dfrac{(-t_in_i/m_i)^{-m_i/2}e^{-t_in_i/2m_i}}{t_in_i\Gamma(m_i/2)\Gamma(n_i/2)(n_i^2-4)}\sum_{r=1}^{4}\gamma_r(t_i;m_i,n_i)$ and

$$\gamma_1(t_i;m_i,n_i)=-m_i(-t_in_i/m_i)^{m_i/2}(-t_in_i/m_i)^{n_i/4}(\Gamma(m_i/2)\Gamma(n_i/2))(n_i+2)(m_i+2)M\left(\frac{m_i}{2}+\frac{n_i}{4}+1,-\frac{1}{2}\left(\frac{n_i}{2}-1\right),\frac{t_in_i}{m_i}\right)$$

$$\gamma_2(t_i;m_i,n_i)=m_i(-t_in_i/m_i)^{(m_i+n_i)/2}(t_in_i/m_i)^{n_i/4}(\Gamma(m_i+n_i/2)\Gamma(-n_i/2))(n_i-2)(m_i+n_i+2)M\left(\frac{m_i}{2}+\frac{n_i}{4}+1,\frac{1}{2}\left(\frac{n_i}{2}+1\right),\frac{t_in_i}{m_i}\right)$$

$$\gamma_3(t_i;m_i,n_i)=(-t_in_i/m_i)^{m_i/2}(t_in_i/m_i)^{n_i/4}(\Gamma(m_i/2)\Gamma(n_i/2))(n_i+2)(n_i(m_i-2t_i)+m_i^2)M\left(\frac{m_i}{2}+\frac{n_i}{4},-\frac{1}{2}\left(\frac{n_i}{2}-1\right),\frac{t_in_i}{m_i}\right)$$

$$\gamma_4(t_i;m_i,n_i)=-(-t_in_i/m_i)^{(m_i+n_i)/2}(t_in_i/m_i)^{n_i/4}(\Gamma(m_i+n_i/2)\Gamma(-n_i/2))(n_i-2)(m_i^2-2n_it_i)M\left(\frac{m_i}{2}+\frac{n_i}{4},\frac{1}{2}\left(\frac{n_i}{2}+1\right),\frac{t_in_i}{m_i}\right)$$

Similarly, $\varphi(t_i;m_i,n_i)=-\dfrac{t_in_i(-t_in_i/m_i)^{-\frac{m_i}{2}}}{m_i\Gamma(m_i/2)}G\left(\left[[0],[]\right],\left[\left[\dfrac{m_i-1}{2},\dfrac{m_i}{2}\right],[]\right],\dfrac{-t_in_i}{m_i}\right)$, M and G are the Whittaker-M function and Meijer-G function respectively.

Theorem 2.3: The Cumulant of the Moment generating function of the Sam-Solai's Multivariate additive F-distribution is

$$C_{x_1,x_2\ldots x_P}(t_1,t_2\ldots t_P)=\sum_{i=1}^{p}\log(\varphi(t_i;m_i,n_i))+\log\left(\sum_{i=1}^{p}\frac{\phi(t_i;m_i,n_i)}{\varphi(t_i;m_i,n_i)}-(p-1)\right) \tag{18}$$

Proof: It is found from $C_{F_1,F_2,F_3,\ldots F_p}(t_1,t_2,t_{3,\ldots}t_p)=\log(M_{F_1,F_2,F_3,\ldots F_p}(t_1,t_2,t_{3,\ldots}t_p))$

Theorem 2.4: The Characteristic function of the Sam-Solai's Multivariate additive F- distribution is

$$\phi_{x_1,x_2\ldots x_P}(t_1,t_2\ldots t_P)=\prod_{j=1}^{p}\psi(t_j;m_j,n_j)\left(\sum_{j=1}^{p}\frac{\omega(t_j;m_j,n_j)}{\psi(t_j;m_j,n_j)}-(p-1)\right) \tag{19}$$

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{F_1,F_2,F_3,\ldots F_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{j=1}^{p}t_jF_j}\,g(F_1,F_2,F_3,\ldots F_p)\prod_{j=1}^{p}dF_j$$

$$\phi_{F_1,F_2,F_3,\ldots F_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{j=1}^{p}t_jF_j}\left\{\left(\sum_{j=1}^{p}\frac{B\left(\frac{m_j}{2},1\right)}{B\left(\frac{m_j}{2},\frac{n_j}{2}\right)}\frac{1}{(1+(m_j/n_j)F_j)^{\frac{n_j}{2}-1}}\right)-(p-1)\right\}\prod_{j=1}^{p}\frac{(m_j/n_j)((m_j/n_j)F_i)^{\frac{m_j}{2}-1}}{B\left(\frac{m_j}{2},1\right)(1+(m_j/n_j)F_j)^{\frac{m_j}{2}+1}}\prod_{j=1}^{p}dF_j \tag{19a}$$

From (19a), it observed

$$\frac{1}{B\left(\frac{m_j}{2},\frac{n_j}{2}\right)}\int_0^\infty e^{it_jF_j}\frac{(m_j/n_j)^{m_j/2}F_j^{\frac{m_j}{2}-1}}{(1+(m_j/n_j)F_j)^{\frac{m_j}{2}+\frac{n_j}{2}}}dF_j=\frac{(-it_jn_j/m_j)^{-m_j/2}e^{-it_jn_j/2m_j}}{it_jn_j\Gamma(m_j/2)\Gamma(n_j/2)(n_j^{\,2}-4)}\sum_{r=1}^{4}\gamma_r(t_j;m_j,n_j)$$

$$\frac{1}{B\left(\frac{m_j}{2},1\right)}\int_0^\infty e^{it_jF_j}\frac{(m_j/n_j)^{m_j/2}F_j^{\frac{m_j}{2}-1}}{(1+(m_j/n_j)F_j)^{\frac{m_j}{2}+1}}dF_j=-\frac{it_jn_j(-it_jn_j/m_j)^{-\frac{m_j}{2}}}{m_j\Gamma(m_j/2)}G\left(\left[[0],[]\right],\left[\left[\frac{m_j-1}{2},\frac{m_j}{2}\right],[]\right],\frac{-it_jn_j}{m_j}\right)$$

Thus, by integration $\phi_{x_1,x_2\ldots x_P}(t_1,t_2\ldots t_P)=\prod_{j=1}^{p}\psi(t_j;m_j,n_j)(\sum_{j=1}^{p}\dfrac{\omega(t_j;m_j,n_j)}{\psi(t_j;m_j,n_j)}-(p-1))$

where $\omega(t_i;m_i,n_i)=\dfrac{(-it_jn_j/m_j)^{-m_j/2}e^{-it_jn_j/2m_j}}{it_jn_j\Gamma(m_j/2)\Gamma(n_j/2)(n_j^{\,2}-4)}\sum_{r=1}^{4}\gamma_r(t_j;m_j,n_j)$ and

$$\gamma_1(t_j;m_j,n_j)=-m_j(-it_jn_j/m_j)^{m_j/2}(-it_jn_j/m_j)^{n_j/4}(\Gamma(m_j/2)\Gamma(n_j/2))(n_j+2)(m_j+2)M\left(\frac{m_j}{2}+\frac{n_j}{4}+1,-\frac{1}{2}\left(\frac{n_j}{2}-1\right),\frac{it_jn_j}{m_j}\right)$$

$$\gamma_2(t_j;m_j,n_j)=m_j(-it_jn_j/m_j)^{(m_j+n_j)/2}(it_jn_j/m_j)^{n_j/4}(\Gamma(m_j+n_j/2)\Gamma(-n_j/2))(n_j-2)(m_j+n_j+2)M\left(\frac{m_j}{2}+\frac{n_j}{4}+1,\frac{1}{2}\left(\frac{n_j}{2}+1\right),\frac{it_jn_j}{m_j}\right)$$

$$\gamma_3(t_j;m_j,n_j)=(-it_jn_j/m_j)^{m_j/2}(it_jn_j/m_j)^{n_j/4}(\Gamma(m_j/2)\Gamma(n_j/2))(n_i+2)(n_j(m_j-2it_j)+m_j^{\,2})M\left(\frac{m_j}{2}+\frac{n_j}{4},-\frac{1}{2}\left(\frac{n_j}{2}-1\right),\frac{it_jn_j}{m_j}\right)$$

$$\gamma_4(t_j;m_j,n_j)=-(-it_jn_j/m_j)^{(m_j+n_j)/2}(it_jn_j/m_j)^{n_j/4}(\Gamma(m_j+n_j/2)\Gamma(-n_j/2))(n_j-2)(m_j^{\,2}-2in_jt_j)M\left(\frac{m_j}{2}+\frac{n_j}{4},\frac{1}{2}\left(\frac{n_j}{2}+1\right),\frac{it_jn_j}{m_j}\right)$$

Similarly, $\psi(t_j;m_j,n_j)=-\frac{it_jn_j(-it_jn_j/m_j)^{-\frac{m_j}{2}}}{m_j\Gamma(m_j/2)}G\left([[0],[]],\left[\left[\frac{m_j-1}{2},\frac{m_j}{2}\right],[]\right],\frac{-it_jn_j}{m_j}\right)$, M and G are the complex Whittaker-M function and complex Meijer-G function respectively.

Theorem 2.5: The survival function of the Sam-Solai's Multivariate additive F- distribution is

$$S(F_1,F_2,F_3\dots,F_p)=1-\left\{\left(\sum_{i=1}^{p}\frac{\phi_i(F_i;m_i,n_i)}{\omega_i(F_i;m_i,n_i)}\right)-(p-1)\right\}\prod_{i=1}^{p}\omega_i(F_i;m_i,n_i) \tag{20}$$

Proof: Let the survival function of a multivariate distribution is given as

$$S(F_1,F_2,F_3\dots,F_p)=1-G(F_1,F_2,F_3\dots,F_p)$$

$$S(F_1,F_2,F_3\dots,F_p)=1-\int_0^{F_1}\int_0^{F_2}\int_0^{F_3}\dots\int_0^{F_p}\left\{\left(\sum_{i=1}^{p}\frac{B(\frac{m_i}{2},1)}{B(\frac{m_i}{2},\frac{n_i}{2})}\frac{1}{(1+(m_i/n_i)u_i)^{\frac{n_i}{2}-1}}\right)-(p-1)\right\}\prod_{i=1}^{p}\frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B(\frac{m_i}{2},1)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}}du_i$$

$$S(F_1,F_2,F_3\dots,F_p)=1-\left\{\left(\sum_{i=1}^{p}\frac{\int_0^{F_i}\frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+\frac{n_i}{2}}}du_i}{\int_0^{F_i}\frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}}du_i}\right)-(p-1)\right\}\prod_{i=1}^{p}\int_0^{F_i}\frac{(m_i/n_i)((m_i/n_i)u_i)^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}}du_i$$

$$S(F_1,F_2,F_3\dots,F_p)=1-\left\{\left(\sum_{i=1}^{p}\frac{\phi_i(F_i;m_i,n_i)}{\omega_i(F_i;m_i,n_i)}\right)-(p-1)\right\}\prod_{i=1}^{p}\omega_i(F_i;m_i,n_i)$$

where $\phi_i(F_i;m_i,n_i)=\int_0^{F_i}\frac{(m_i/n_i)^{\frac{m_i}{2}}u_i^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},\frac{n_i}{2}\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+\frac{n_i}{2}}}du_i$ and $\omega_i(F_i;m_i,n_i)=\int_0^{F_i}\frac{(m_i/n_i)^{\frac{m_i}{2}}u_i^{\frac{m_i}{2}-1}}{B\left(\frac{m_i}{2},1\right)(1+(m_i/n_i)u_i)^{\frac{m_i}{2}+1}}du_i$ are the lower incomplete Beta integrals of i^{th} F-variables

Theorem 2.6: The hazard function of the Sam-Solai's Multivariate additive F- distribution is

$$h(F_1,F_2,F_3\dots,F_p)=\frac{\{(\sum_{i=1}^{p}\frac{B(\frac{m_i}{2},1)}{B(\frac{m_i}{2},\frac{n_i}{2})}\frac{1}{(1+(m_i/n_i)F_i)^{\frac{n_i}{2}-1}})-(p-1)\}\prod_{i=1}^{p}\frac{(m_i/n_i)((m_i/n_i)F_i)^{\frac{m_i}{2}-1}}{B(\frac{m_i}{2},1)(1+(m_i/n_i)F_i)^{\frac{m_i}{2}+1}}}{1-\{(\sum_{i=1}^{p}\frac{\phi_i(F_i;m_i,n_i)}{\omega_i(F_i;m_i,n_i)})-(p-1)\}\prod_{i=1}^{p}\omega_i(F_i;m_i,n_i)} \tag{21}$$

Proof: It is obtained from

$$h(F_1,F_2,F_3\dots,F_p)=\frac{g(F_1,F_2,F_3\dots,F_p)}{S(F_1,F_2,F_3\dots,F_p)} \text{ and } S(F_1,F_2,F_3\dots,F_p)=1-G(F_1,F_2,F_3\dots,F_p)$$

Theorem 2.7: The Cumulative hazard function of the Sam-Solai's Multivariate additive F- distribution is

$$H(F_1,F_2,F_3\dots,F_p)=-\log\left(1-\left\{\left(\sum_{i=1}^{p}\frac{\phi_i(F_i;m_i,n_i)}{\omega_i(F_i;m_i,n_i)}\right)-(p-1)\right\}\prod_{i=1}^{p}\omega_i(F_i;m_i,n_i)\right) \tag{22}$$

Proof: Let the Cumulative hazard function of a multivariate distribution is given as

$$H(F_1, F_2, F_3 \ldots, F_p) = -\log(1 - G(F_1, F_2, F_3 \ldots, F_p))$$

$$H(F_1, F_2, F_3 \ldots, F_p) = -\log(S(F_1, F_2, F_3 \ldots, F_p))$$

$$H(F_1, F_2, F_3 \ldots, F_p) = -\log\left(1 - \left\{\left(\sum_{i=1}^{p} \frac{\phi_i(F_i; m_i, n_i)}{\omega_i(F_i; m_i, n_i)}\right) - (p-1)\right\} \prod_{i=1}^{p} \omega_i(F_i; m_i, n_i)\right)$$

SECTION 3: SOME SPECIAL CASES

Results 3.1: The uni-variate marginal of the Sam-Solai's multivariate additive F- distribution of kind-2 are the uni-variate F-distributions.

Result 3.2*:* From (1) and if ***P*=1**, then the Sam-Solai's multivariate additive F- density is reduced to density of univariate F-distribution.

Result 3.3: From (1) and if ***P*=2**, then the density of Sam-Solai's Multivariate F- distribution of Kind-2 was reduced into

$$g(F_1, F_2) = \left\{ \frac{B\left(\frac{m_1}{2}, 1\right)}{B\left(\frac{m_1}{2}, \frac{n_1}{2}\right)(1 + (m_1/n_1)F_1)^{\frac{n_1}{2}-1}} + \frac{B\left(\frac{m_2}{2}, 1\right)}{B\left(\frac{m_2}{2}, \frac{n_2}{2}\right)(1 + (m_2/n_2)F_2)^{\frac{n_2}{2}-1}} - 1 \right\} \frac{(m_1/n_1)^{\frac{m_1}{2}} F_1^{\frac{m_1}{2}-1}}{B\left(\frac{m_1}{2}, 1\right)(1 + (m_1/n_1)F_1)^{\frac{m_1}{2}+1}} \frac{(m_2/n_2)^{\frac{m_2}{2}} F_2^{\frac{m_2}{2}-1}}{B\left(\frac{m_2}{2}, 1\right)(1 + (m_2/n_2)F_2)^{\frac{m_2}{2}+1}} \quad (23)$$

where $0 \le F_1, F_2 \le \infty$, $m_1, m_2, n_1, n_2 > 0$

This is called Sam-Solai's Bi-variate additive F- distribution of kind-2

Result 3.4: The table 1 and Bi-variate probability surface for (23) shows the selected simulated standard Bi-variate (Negative) correlations between two F-variables which are bounded between 0 and -1 calculated from 10,000 different combinations for m_1, n_1, m_2, n_2 d.f.

Table 1: ***Simulation of Bi-variate F- correlations for different combinations of d.f***

Runs	m_1	n_1	m_2	n_2	$\rho(x_1, x_2)$
3416	***0.50***	***1.70***	***0.10***	***2.90***	***-0.9941***
8582	***3.30***	***1.70***	***0.10***	***2.50***	
1139	*2.10*	*0.50*	*0.90*	*1.70*	*-0.9029*
6648	*3.70*	*0.90*	*2.10*	*0.10*	*-0.8000*
4278	***2.90***	***0.10***	***2.10***	***0.50***	***-0.7006***
8267	*1.70*	*0.50*	*0.50*	*3.30*	*-0.6013*
6888	***3.30***	***1.70***	***0.50***	***2.10***	***-0.5004***
3146	*0.90*	*1.30*	*3.30*	*0.10*	*-0.4003*
9226	*2.90*	*3.30*	*1.30*	*0.90*	*-0.3001*
3954	*2.90*	*0.50*	*0.90*	*3.30*	*-0.2001*
8483	***3.30***	***0.50***	***2.90***	***2.90***	***-0.1000***
8603	***2.90***	***0.90***	***0.90***	***3.70***	
8103	***0.10***	***3.70***	***3.70***	***3.70***	***-0.0020***

$m_1 = 0.5\ n_1 = 1.7\ m_2 = 0.1\ n_2 = 2.9\ \ \rho(F_1, F_2) = -0.9941$

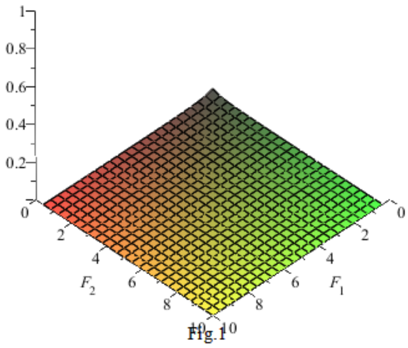

Fig.1

$m_1 = 3.3\ n_1 = 1.7\ m_2 = 0.1\ n_2 = 2.5\ \ \rho(F_1, F_2) = -0.9941$

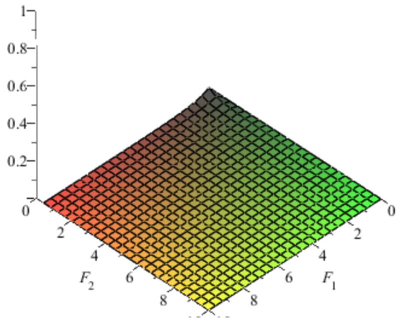

Fig.2

$m_1 = 2.9\ n_1 = 0.1\ m_2 = 2.1\ n_2 = 0.5\ \ \rho(F_1, F_2) = -0.7006$

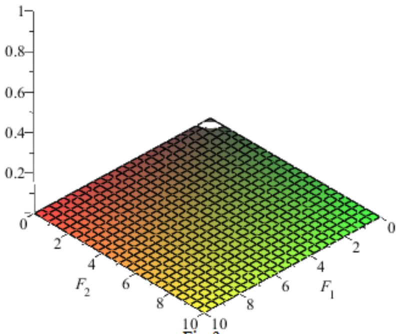

Fig.3

$m_1 = 3.3\ n_1 = 1.7\ m_2 = 0.5\ n_2 = 2.1\ \ \rho(F_1, F_2) = -0.5004$

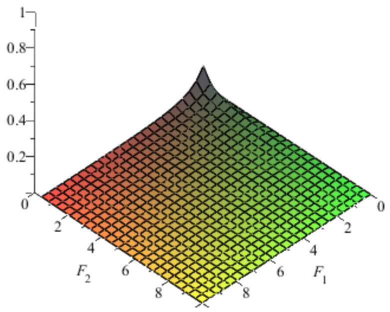

Fig.4

$m_1 = 3.3\ n_1 = 0.5\ m_2 = 2.9\ n_2 = 2.9\ \ \rho(F_1, F_2) = -0.1000$

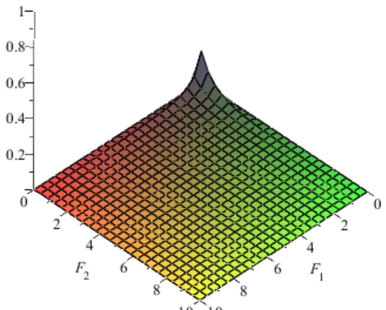

Fig.5

$m_1 = 2.9\ n_1 = 0.9\ m_2 = 0.9\ n_2 = 3.7\ \ \rho(F_1, F_2) = -0.1000$

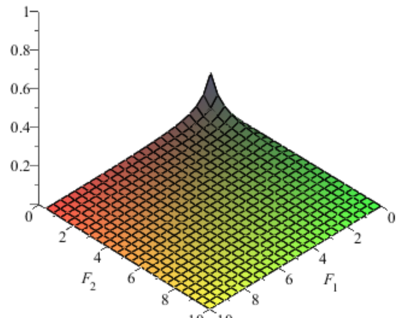

Fig.6

$m_1 = 0.1\ n_1 = 3.7\ m_2 = 3.7\ n_2 = 3.7\ \ \rho(F_1, F_2) = -0.0020$

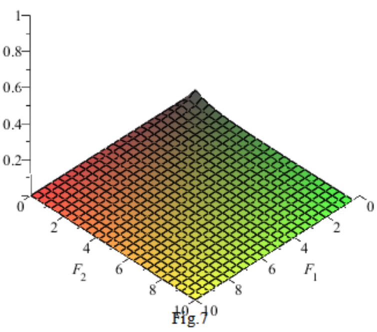

Fig.7

Result:3.5 From(1) and if $F_i^{'} = 1/F_i$,then the Sam-solai's Multivariate additive F- distribution of Kind-2 with (m_i, n_i) d.f transformed into same Sam-solai's Multivariate additive $F^{'}$-distribution of Kind-2 with (n_i, m_i) d.f and its density function is given as

$$g(F_1, F_2, F_3 \ldots, F_p) = \left\{ \left(\sum_{i=1}^{p} \frac{B\left(\frac{m_i}{2}, 1\right)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{((n_i / m_i) F_i^{'})^{\frac{n_i}{2}-1}}{(1 + (n_i / m_i) F_i^{'})^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{(n_i / m_i)}{B\left(\frac{m_i}{2}, 1\right)(1 + (n_i / m_i) F_i^{'})^{\frac{m_i}{2}+1}} \tag{24}$$

where $0 \le F_i^{'} < +\infty$, $n_i, m_i > 0$

Result:3.6 From(1) and if $x_i = (m_i / n_i) F_i / (1 + (m_i / n_i) F_i)$, $m_i = 2a_i, n_i = 2b_i$ then the density of Sam-Solai's Multivariate additive F- distribution of Kind-2 transformed into Sam-Solai's Multivariate additive Beta distribution of Kind-1 of Type-B with shape parameters (a_i, b_i) and its density function is given as

$$g(x_1, x_2, x_3 \ldots, x_p) = \left\{ \left(\sum_{i=1}^{p} \frac{(1-x_i)^{b_i - 1} B(a_i, 1)}{B(a_i, b_i)} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{x_i^{a_i - 1}}{B(a_i, 1)} \tag{25}$$

where $0 \le x_i < 1 \quad a_i, b_i > 0$

Result:3.7 From (1) and if $x_i = m_i F_i / n_i$, $m_i = 2a_i, n_i = 2b_i$,then the Sam-solai's Multivariate additive F- distribution of Kind-2 transformed into Sam-solai's Multivariate additive Beta distribution of Kind-2 of Type-Awith shape parameters (a_i, b_i) and its density function is given as

$$g(x_1, x_2, x_3 \ldots, x_p) = \left\{ \left(\sum_{i=1}^{p} \frac{B(a_i, 1)}{B(a_i, b_i)} \frac{1}{(1+x_i)^{b_i - 1}} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{x_i^{a_i - 1}}{B(a_i, 1)(1+x_i)^{a_i + 1}} \tag{26}$$

where $0 \le x_i \le \infty \quad a_i, b_i > 0$

Result:3.8 From (1) and if $Z_i = \log F_i / 2$, then the Sam-solai's Multivariate additive F- distribution of Kind-1 transformed into Sam-solai's generalization of Multivariate Fisher's Z distribution of Kind-1 with (m_i, n_i) d.f and its density function is given as

$$g(F_1, F_2, F_3 \ldots, F_p) = \left\{ \left(\sum_{i=1}^{p} \frac{B(\frac{m_i}{2}, 1)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1 + (m_i / n_i) e^{2Z_i})^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} 2^P \prod_{i=1}^{p} \frac{((m_i / n_i) e^{2Z_i})^{\frac{m_i}{2}}}{B\left(\frac{m_i}{2}, 1\right)(1 + (m_i / n_i) e^{2Z_i})^{\frac{m_i}{2}+1}} \tag{27}$$

where $-\infty \le Z_i \le +\infty$, $m_i, n_i > 0$

Result:3.9 From(1) and if $x_i = -\log F_i$, then the Sam-solai's Multivariate additive F- distribution of Kind-1 transformed into Sam-solai's generalization of Multivariate Logistic- F distribution of Kind-1 with parameters (m_i, n_i) and its density function is given as

$$g(x_1, x_2, x_3 \ldots, x_p) = \left\{ \left(\sum_{i=1}^{p} \frac{B\left(\frac{m_i}{2}, 1\right)}{B\left(\frac{m_i}{2}, \frac{n_i}{2}\right)} \frac{1}{(1 + (m_i / n_i) e^{-x_i})^{\frac{n_i}{2}-1}} \right) - (p-1) \right\} \prod_{i=1}^{p} \frac{((m_i / n_i) e^{-x_i})^{\frac{m_i}{2}}}{B\left(\frac{m_i}{2}, 1\right)(1 + (m_i / n_i) e^{-x_i}{}_i)^{\frac{m_i}{2}+1}} \tag{28}$$

where $-\infty \le x_i \le +\infty$, $m_i, n_i > 0$

CONCLUSION

The multivariate generalization of F- distribution in an additive form of Sam-Solai's generalization having some interesting features. At first, the marginal univariate distributions of the Sam-Solai's Multivariate additive F-distribution are univariate and enjoyed the symmetric property. Secondly, the Correlation co-efficient between any two F-variables of the proposed distribution is bounded between -1 and +1 for certain Fractional d.f and the authors established the simulated standard bivariate correlations. Finally, the multivariate generalization of F-distribution in an additive form open the way for the same additive form of the transformation of Sam-Solai's Multivariate additive F-distribution of kind-2 into Multivariate additive Beta distribution of Kind-1 of Type-B, Multivariate Beta-distribution of Kind-2 of Type-B, Multivariate Fisher's Z-distribution of Kind-2 and Multivariate Logistic-F distribution of kind-2.

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