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A NEW GENERALISATION OF SAM-SOLAI'S MULTIVARIATE ADDITIVE EXPONENTIAL DISTRIBUTION*

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ABSTRACT

This paper proposed a new generalization of bounded Continuous multivariate symmetric probability distributions. More specifically the authors visualizes a new generalization of Sam-Solai's Multivariate additive Exponential distribution from the uni-variate exponential distribution.Further,we find its Marginal, Multivariate Conditional distributions, Multivariate Generating functions, Multivariate survival, hazard functions and also discussed it's special cases. The special cases includes the transformation of Sam-Solai's Multivariate additive exponential distribution into Multivariate Inverse exponential distribution, Multivariate Weibull distribution, Multivariate Power law distribution, Multivariate chi-square distribution with two d.f, Multivariate Rayleigh distribution, Multivariate Pareto distribution, Multivariate logistic distribution, Multivariate Generalized extreme value distribution and Multivariate Benktander weibull distribution. Moreover, the bivariate correlation between any two exponential random variables found to be -0.25 and it is independent from the Co-variance. Similarly, we simulated and established a symmetric matrix of Co-variances based on different combinations of values for parameters.

Keywords: *Sam-Solai's Multivariate Exponential distribution, Multivariate Inverse exponential distribution, Multivariate Weibull distribution, Multivariate Power law distribution, Multivariate chi-square distribution, Multivariate Rayleigh distribution, Multivariate Pareto distribution, Multivariate logistic distribution, Multivariate Generalized extreme value distribution and Multivariate Benktander Weibull distribution.*

Mathematics Subject Classification: *Primary 62H10; Secondary 62E15.*

INTRODUCTION:

The exponential distribution was extensively studied in the past and it is used in the field of Reliability theory, queuing theory and also applied to test the life length of equipment, machineries etc. In recent times, statisticians showed the interest to generalize the univariate distributions to its multivariate case and exponential distribution is not an exception to its multivariate generalization. Some authors gave more concentration to the bivariate generalization of exponential distribution with different assumptions respective to their fields. Gumbel (1960), Freud (1961) Marshal and Olkin (1967), Downton(1970), Block et al(1975,1975b,1977), Fridayandpatil (1977), Raftery(1984), Cowan(1987), Sarkar (1987) Arnold and Strauss(1988), Hayakawa(1994) and Kotz et al(1999) were extensively studied the Bivariate generalization of exponential distribution and give different forms and shapes of Bivariate exponential distribution. Similarly these authors also attempted to give the multivariate generalization of the exponential distribution. Krishnamoorthy and Parathsarathy(1951) studied the multivariate exponential distribution as a special case of a multivariate Gamma type distribution and Gumbel (1961) proposed a new form of multivariate exponential distribution. Likwise Marshalland Olkin (1967a, 1995), Essary(1974), Block (1975a), Raftery(1994), Lindleyand s I n g p-urwalla (1986), Ghurye(1987),O'cinneide. et.al (1989),singpurwalla and youngren (1993) introduced new form of multivariate exponential distribution and some authors proposed alternate form of multivariate exponential distribution and applied it to the specific field of application respectively. This paper deals with the new and alternate generalization of multivariate exponential distribution and the authors discussed its properties in the next section.

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SECTION 1: SAM-SOLAI'S MULTIVARIATE ADDITIVE EXPONENTIAL DISTRIBUTION

Definition 1.1: Let $X_1, X_2, X_3 ..., X_p$ be the random variables followed Continuous univariate exponential distribution with parameter λ_i for all i (*i=1 to p*), then the density function of the Multivariate Sam-Solai's additive exponential distribution is defined as

$$f(x_1, x_2, x_3 ..., x_p) = \{(2\sum_{i=1}^{p} e^{-\lambda_i x_i}) - (2^p e^{-\sum_{i=1}^{p}\lambda_i x_i}) - (p-2)\}(\prod_{i=1}^{p}\lambda_i) e^{-\sum_{i=1}^{p}\lambda_i x_i} \quad (1)$$

where $0 \le x_i < \infty \quad \lambda_i > 0$

Theorem 1.2: The cumulative distribution function of the Sam's Multivariate additive exponential distribution is defined by

$$F(x_1, x_2, x_3 ..., x_p) = \int_0^{x_1}\int_0^{x_2}\int_0^{x_3}...\int_0^{x_p} \{(2\sum_{i=1}^{p} e^{-\lambda_i u_i}) - (2^p e^{-\sum_{i=1}^{p}\lambda_i u_i}) - (p-2)\}(\prod_{i=1}^{p}\lambda_i) e^{-\sum_{i=1}^{p}\lambda_i u_i} du_1 du_2 du_3 ... du_p \quad (2)$$

where $0 \le u_i < x_i \quad \lambda_i > 0$

$$F(x_1, x_2, x_3 ..., x_p) = \prod_{i=1}^{p}(1 - e^{-\lambda_i x_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})} - \prod_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})} - (p-2)\}$$

Theorem 1.3: The Probability density function of Sam-Solai's Multivariate additive Conditional exponential distribution of X_1 on $X_2, X_3, ... X_p$ is

$$f(x_1 / x_2, x_3 ..., x_p) = \frac{\lambda_1 e^{-\lambda_1 x_1}\{(2\sum_{i=1}^{p} e^{-\lambda_i x_i}) - (2^p e^{-\sum_{i=1}^{p}\lambda_i x_i}) - (p-2)\}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}} \quad (3)$$

where $0 \le x_1 < \infty \quad \lambda_1 > 0$

Proof: It is obtained from
$$f(x_1 / x_2, x_3 ..., x_p) = \frac{f(x_1, x_2, x_3 ..., x_p)}{f(x_2, x_3 ..., x_p)}$$

Theorem 1.4: Mean and Variance of Sam - Solai's Multivariate additive Conditional exponential distribution are

$$E(x_1 / x_2, x_3 ..., x_p) = \frac{\frac{1}{\lambda_1}\{(\frac{1}{2} + 2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-2} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-2\ \}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}} \quad (4)$$

$$V(x_1 / x_2, x_3 ..., x_p) = E(x_1^2 / x_2, x_3 ..., x_p) - (E(x_1 / x_2, x_3 ..., x_p))^2 \quad (5)$$

where
$$E(x_1^2 / x_2, x_3 ..., x_p) = \frac{\frac{2}{\lambda_1^2}\{(\frac{1}{4} + 2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-3} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-2\ \}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}}$$

Proof: The n^{th} order moment of the distribution is

$$E(x_1^n / x_2, x_3 ..., x_p) = \int_0^{\infty} x_1^n f(x_1 / x_2, x_3 ..., x_p) dx_1$$

$$E(x_1^n / x_2, x_3 \ldots, x_p) = \int_0^\infty x_1^n \frac{\lambda_1 e^{-\lambda_1 x_1}\{(2\sum_{i=1}^{p} e^{-\lambda_i x_i}) - (2^p e^{-\sum_{i=1}^{p}\lambda_i x_i}) - (p-2)\}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}} dx_1$$

$$E(x_1^n / x_2, x_3 \ldots, x_p) = \frac{\frac{1}{\lambda_1^n}\{\frac{\Gamma(n+k_1)}{\Gamma k_1} + \Gamma(n+1)\{(\sum_{i=2}^{p}\frac{(\lambda_i x_i)^{k_i-1}}{\Gamma k_i}) - (p-1)\}\}}{(\sum_{i=2}^{p}\frac{(\lambda_i x_i)^{k_i-1}}{\Gamma k_i}) - (p-2))}$$

If ***n=1***, then the Conditional expectation is

$$E(x_1 / x_2, x_3 \ldots, x_p) = \frac{\frac{1}{\lambda_1}\{(\frac{1}{2} + 2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-2} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-2\ \}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}}$$

If ***n=2***, then the second order moment is

$$E(x_1^2 / x_2, x_3 \ldots, x_p) = \frac{\frac{2}{\lambda_1^2}\{(\frac{1}{4} + 2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-3} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-2\ \}}{\{(2\sum_{i=2}^{p} e^{-\lambda_i x_i}) - (2^{p-1} e^{-\sum_{i=2}^{p}\lambda_i x_i}) - (p-3)\}}$$

The conditional variance of the distribution is obtained by Substituting the first and second moments in (5).

Theorem 1.5: If there are ***p = (q + k)*** random variables, such that ***q*** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depends on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$, then the density function of Sam-Solai's multivariate additive conditional Exponential distribution is

$$f(x_1, x_2, x_3 \ldots, x_q / x_{q+1}, x_{q+2}, x_{q+3} \ldots, x_{q+k}) = \frac{(\prod_{i=1}^{q}\lambda_i) e^{-\sum_{i=1}^{q}\lambda_i x_i}\{(2\sum_{i=1}^{q+k} e^{-\lambda_i x_i}) - (2^{q+k} e^{-\sum_{i=1}^{q+k}\lambda_i x_i}) - (q+k-2)\}}{\{(2\sum_{i=q+1}^{q+k} e^{-\lambda_i x_i}) - (2^k e^{-\sum_{i=q+1}^{q+k}\lambda_i x_i}) - (k-2)\}} \tag{6}$$

where $0 \le x_i < \infty \quad \lambda_i > 0$

Proof: Let the multivariate conditional law for ***q*** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depending on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$ is given as

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{f(x_1, x_2, x_3, \ldots x_q, x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}{f(x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{(\prod_{i=1}^{q+k}\lambda_i) e^{-\sum_{i=1}^{q+k}\lambda_i x_i}\{(2\sum_{i=1}^{q+k} e^{-\lambda_i x_i}) - (2^{q+k} e^{-\sum_{i=1}^{q+k}\lambda_i x_i}) - (q+k-2)\}}{\int_0^\infty\int_0^\infty \ldots \int_0^\infty (\prod_{i=1}^{q+k}\lambda_i) e^{-\sum_{i=1}^{q+k}\lambda_i x_i}\{(2\sum_{i=1}^{q+k} e^{-\lambda_i x_i}) - (2^{q+k} e^{-\sum_{i=1}^{q+k}\lambda_i x_i}) - (q+k-2)\}\prod_{i=1}^{q} dx_i}$$

$$f(x_1, x_2, x_3 \ldots, x_q / x_{q+1}, x_{q+2}, x_{q+3} \ldots, x_{q+k}) = \frac{(\prod_{i=1}^{q}\lambda_i) e^{-\sum_{i=1}^{q}\lambda_i x_i}\{(2\sum_{i=1}^{q+k} e^{-\lambda_i x_i}) - (2^{q+k} e^{-\sum_{i=1}^{q+k}\lambda_i x_i}) - (q+k-2)\}}{\{(2\sum_{i=q+1}^{q+k} e^{-\lambda_i x_i}) - (2^k e^{-\sum_{i=q+1}^{q+k}\lambda_i x_i}) - (k-2)\}}$$

where $0 \le x_i < \infty \quad \lambda_i > 0$

SECTION 2: CONSTANTS OF SAM-SOLAI'S MULTIVARIATE ADDITIVE EXPONENTIAL DISTRIBUTION

Theorem 2.1: The Marginal product moments, Co-variance and Population Correlation Co-efficient between the Exponential random variables X_1 and X_2 are given as

$$E(x_1x_2)=\frac{3}{4\lambda_1\lambda_2} \tag{7}$$

$$COV(x_1,x_2)=-\frac{1}{4\lambda_1\lambda_2} \tag{8}$$

$$\rho(x_1,x_2)=-\frac{1}{4} \tag{9}$$

Proof: Assume that X_1 and X_2 are random variables from Sam-Solai's multivariate additive exponential distribution. Let the product moment of the distribution is

$$E(x_1x_2)=\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} x_1x_2\, f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^{p} d\,x_i$$

Its Co-variance is

$$COV(x_1,x_2)=E(x_1x_2)-E(x_1)E(x_2) \tag{10}$$

Then

$$E(x_1x_2)=\int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} x_1x_2\{(2\sum_{i=1}^{p}e^{-\lambda_i x_i})-(2^p e^{-\sum_{i=1}^{p}\lambda_i x_i})-(p-2)\}(\prod_{i=1}^{p}\lambda_i)e^{-\sum_{i=1}^{p}\lambda_i x_i}\prod_{i=1}^{p} d\,x_i$$

By evaluation, it follows that $E(x_1x_2)=\dfrac{3}{4\lambda_1\lambda_2}$

The Marginal expectation of exponential variables X_1 and X_2 are $1/\lambda_1$ and $1/\lambda_2$ respectively. The Marginal Product moment for $E(x_1x_2)$ is obtained by substituting the above Marginal expectations for X_1 and X_2 in (10).
Thus

$$COV(x_1,x_2)=-\frac{1}{4\lambda_1\lambda_2} \tag{11}$$

Correlation coefficient of a distribution is (12a)

$$\rho(x_1,x_2)=\frac{COV(x_1,x_2)}{\sigma_1\sigma_2}$$

It observed that $\sigma_1 = 1/\lambda_1$ and $\sigma_2 = 1/\lambda_2$ (12b)

From (11), (12a) and (12b), it follows that

$$\rho(x_1,x_2)=-\frac{1}{4} \tag{13}$$

Remark 2.1*:* The Product moments, Co-variance and population Correlation Co-efficient between the i^{th} and j^{th} of Sam-Solai's multivariate additive exponential distribution random variable are given as

$$E(x_ix_j)=\frac{3}{4\lambda_i\lambda_j} \tag{14}$$

$$COV(x_i,x_j)=-\frac{1}{4\lambda_i\lambda_j} \tag{15}$$

$$\rho(x_i,x_j)=-\frac{1}{4} \tag{16}$$

Theorem 2.2: The Moment generating function of Sam-Solai's Multivariate additive exponential distribution is

$$M_{x_1,x_2,x_3\ldots,x_p}(t_1,t_2,t_3\ldots,t_p)=(\prod_{i=1}^{p}\frac{\lambda_i}{\lambda_i-t_i})\{2\sum_{i=1}^{p}\frac{(\lambda_i-t_i)}{(2\lambda_i-t_i)}-(2^p\prod_{i=1}^{p}\frac{\lambda_i-t_i}{2\lambda_i-t_i})-(p-2)\} \tag{17}$$

Proof: Let the moment generating function of a Multivariate distribution is given as

$$M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{\sum_{i=1}^{p}t_ix_i}\; f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^{p}dx_i$$

$$M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{\sum_{i=1}^{p}t_ix_i}\;\{(2\sum_{i=1}^{p}e^{-\lambda_ix_i})-(2^pe^{-\sum_{i=1}^{p}\lambda_ix_i})-(p-2)\}(\prod_{i=1}^{p}\lambda_i)e^{-\sum_{i=1}^{p}\lambda_ix_i}\prod_{i=1}^{p}dx_i$$

$$M_{x_1,x_2,x_3\ldots,x_p}(t_1,t_2,t_3\ldots,t_p)=(\prod_{i=1}^{p}\frac{\lambda_i}{\lambda_i-t_i})\{2\sum_{i=1}^{p}\frac{(\lambda_i-t_i)}{(2\lambda_i-t_i)}-(2^p\prod_{i=1}^{p}\frac{\lambda_i-t_i}{2\lambda_i-t_i})-(p-2)\}$$

by integrating the above equation.

Theorem 2.3: The Cumulant of the Moment generating function of the Sam-Solai's Multivariate additive exponential distribution is

$$C_{x_1,x_2,x_3\ldots,x_p}(t_1,t_2,t_3\ldots,t_p)=\sum_{i=1}^{p}\log(\frac{\lambda_i}{\lambda_i-t_i})+\log\{2\sum_{i=1}^{p}\frac{(\lambda_i-t_i)}{(2\lambda_i-t_i)}-(2^p\prod_{i=1}^{p}\frac{\lambda_i-t_i}{2\lambda_i-t_i})-(p-2)\} \tag{18}$$

Proof: It is found from

$$C_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)=\log(M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)).$$

Theorem 2.4: The Characteristic function of the Sam-Solai's Multivariate additive exponential distribution is

$$\phi_{x_1,x_2,x_3\ldots,x_p}(t_1,t_2,t_3\ldots,t_p)=(\prod_{j=1}^{p}\frac{\lambda_j}{\lambda_j-it_j})\{2\sum_{j=1}^{p}\frac{(\lambda_j-it_j)}{(2\lambda_j-it_j)}-(2^p\prod_{j=1}^{p}\frac{\lambda_j-it_j}{2\lambda_j-t_j})-(p-2)\} \tag{19}$$

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{j=1}^{p}t_jx_j}\; f(x_1,x_2,x_3,\ldots x_p)\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{j=1}^{p}t_jx_j}\;\{(2\sum_{j=1}^{p}e^{-\lambda_jx_j})-(2^pe^{-\sum_{j=1}^{p}\lambda_jx_j})-(p-2)\}(\prod_{j=1}^{p}\lambda_j)e^{-\sum_{j=1}^{p}\lambda_jx_j}\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2,x_3\ldots,x_p}(t_1,t_2,t_3\ldots,t_p)=(\prod_{j=1}^{p}\frac{\lambda_j}{\lambda_j-it_j})\{2\sum_{j=1}^{p}\frac{(\lambda_j-it_j)}{(2\lambda_j-it_j)}-(2^p\prod_{j=1}^{p}\frac{\lambda_j-it_j}{2\lambda_j-t_j})-(p-2)\}$$

by integrating the above equation.

Theorem 2.5: The survival function of the Sam-Solai's Multivariate additive exponential distribution is

$$S(x_1,x_2,x_3\ldots,x_p)=1-\prod_{i=1}^{p}(1-e^{-\lambda_ix_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_ix_i})}{(1-e^{-\lambda_ix_i})}-\prod_{i=1}^{p}\frac{(1-e^{-2\lambda_ix_i})}{(1-e^{-\lambda_ix_i})}-(p-2)\} \tag{20}$$

Proof: Let the survival function of a multivariate distribution is given as

$$S(x_1,x_2,x_3\ldots,x_p)=1-F(x_1,x_2,x_3\ldots,x_p)$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-\int_0^{x_1}\int_0^{x_2}\int_0^{x_3}\cdots\int_0^{x_p}\{(2\sum_{i=1}^{p}e^{-\lambda_i u_i})-(2^p e^{-\sum_{i=1}^{p}\lambda_i u_i})-(p-2)\}(\prod_{i=1}^{p}\lambda_i)e^{-\sum_{i=1}^{p}\lambda_i u_i}du_1du_2du_3\ldots du_p$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-\prod_{i=1}^{p}(1-e^{-\lambda_i x_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-\prod_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-(p-2)\}$$

Theorem 2.6: The hazard function of the Sam-Solai's Multivariate additive exponential distribution is

$$h(x_1,x_2,x_3\ldots,x_p)=\frac{\{(2\sum_{i=1}^{p}e^{-\lambda_i x_i})-(2^p e^{-\sum_{i=1}^{p}\lambda_i x_i})-(p-2)\}(\prod_{i=1}^{p}\lambda_i)e^{-\sum_{i=1}^{p}\lambda_i x_i}}{1-\prod_{i=1}^{p}(1-e^{-\lambda_i x_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-\prod_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-(p-2)\}} \tag{21}$$

Proof: It is obtained from

$$h(x_1,x_2,x_3\ldots,x_p)=\frac{f(x_1,x_2,x_3\ldots,x_p)}{S(x_1,x_2,x_3\ldots,x_p)} \text{ and}$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-F(x_1,x_2,x_3\ldots,x_p)$$

Theorem 2.7: The Cumulative hazard function of the Sam-Solai's Multivariate additive exponential distribution is

$$H(x_1,x_2,x_3\ldots,x_p)=-\log(1-\prod_{i=1}^{p}(1-e^{-\lambda_i x_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-\prod_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-(p-2)\}) \tag{22}$$

Proof: Let the Cumulative hazard function of a multivariate distribution is given as

$$H(x_1,x_2,x_3\ldots,x_p)=-\log(1-F(x_1,x_2,x_3\ldots,x_p)$$

$$H(x_1,x_2,x_3\ldots,x_p)=-\log(S(x_1,x_2,x_3\ldots,x_p))$$

$$H(x_1,x_2,x_3\ldots,x_p)=-\log(1-\prod_{i=1}^{p}(1-e^{-\lambda_i x_i})\{\sum_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-\prod_{i=1}^{p}\frac{(1-e^{-2\lambda_i x_i})}{(1-e^{-\lambda_i x_i})}-(p-2)\})$$

SECTION 3: SOME SPECIAL CASES

Result 3.1: The uni-variate marginal of the Sam-Solai's multivariate additive exponential distribution is the uni-variate two parameter exponential distributions.

Result 3.2: From (1) and If ***P*= 1**, the Sam-Solai's multivariate additive exponential density is reduced to density of univariate exponential distribution.

Result 3.3: From (1) If ***P*=2**, then the density of Sam-Solai's Multivariate Exponential distribution was reduced into

$$f(x_1,x_2)=(2e^{-\lambda_1 x_1}+2e^{-\lambda_2 x_2}-4e^{-(\lambda_1 x_1+\lambda_2 x_2)})\lambda_1\lambda_2 e^{-(\lambda_1 x_1+\lambda_2 x_2)} \tag{23}$$

where $0\le x_1,x_2<\infty$, $\lambda_1,\lambda_2>0$

This is called the density of Sam-Solai's Bi-variate additive Exponential distribution.

Result 3.4: Table 1 and Bi-variate probability surface for (23) show the selected simulated standard Bi-variate Co-variances between any two exponential random variables calculated for different combinations of parameters values (λ_i,λ_j).

Table1: Standard Co-variances between two exponential random variables for different values of parameters λ_i and λ_j

λ_i / λ_j	***0.1***	***0.5***	***1***	***1.5***	***2***	***2.5***	***3***	***3.5***	***4***	***4.5***	***5***	***5.5***	***6***	***6.5***	***7***
0.1	.00	-.01	-.03	-.04	-.05	-.06	-.08	-.09	-.10	-.11	-.13	-.14	-.15	-.16	-.18
0.5	-.01	-.06	-.13	-.19	-.25	-.31	-.38	-.44	-.50	-.56	-.63	-.69	-.75	-.81	-.88
1	-.03	-.13	-.25	-.38	-.50	-.63	-.75	-.88	-1.00	-1.13	-1.25	-1.38	-1.50	-1.63	-1.75
1.5	-.04	-.19	-.38	-.56	-.75	-.94	-1.13	-1.31	-1.50	-1.69	-1.88	-2.06	-2.25	-2.44	-2.63
2	-.05	-.25	-.50	-.75	-1.00	-1.25	-1.50	-1.75	-2.00	-2.25	-2.50	-2.75	-3.00	-3.25	-3.50
2.5	-.06	-.31	-.63	-.94	-1.25	-1.56	-1.88	-2.19	-2.50	-2.81	-3.13	-3.44	-3.75	-4.06	-4.38
3	-.08	-.38	-.75	-1.13	-1.50	-1.88	-2.25	-2.63	-3.00	-3.38	-3.75	-4.13	-4.50	-4.88	-5.25
3.5	-.09	-.44	-.88	-1.31	-1.75	-2.19	-2.63	-3.06	-3.50	-3.94	-4.38	-4.81	-5.25	-5.69	-6.13
4	-.10	-.50	-1.00	-1.50	-2.00	-2.50	-3.00	-3.50	-4.00	-4.50	-5.00	-5.50	-6.00	-6.50	-7.00
4.5	-.11	-.56	-1.13	-1.69	-2.25	-2.81	-3.38	-3.94	-4.50	-5.06	-5.63	-6.19	-6.75	-7.31	-7.88
5	-.13	-.63	-1.25	-1.88	-2.50	-3.13	-3.75	-4.38	-5.00	-5.63	-6.25	-6.88	-7.50	-8.13	-8.75
5.5	-.14	-.69	-1.38	-2.06	-2.75	-3.44	-4.13	-4.81	-5.50	-6.19	-6.88	-7.56	-8.25	-8.94	-9.63
6	-.15	-.75	-1.50	-2.25	-3.00	-3.75	-4.50	-5.25	-6.00	-6.75	-7.50	-8.25	-9.00	-9.75	-10.50
6.5	-.16	-.81	-1.63	-2.44	-3.25	-4.06	-4.88	-5.69	-6.50	-7.31	-8.13	-8.94	-9.75	-10.56	-11.38
7	-.18	-.88	-1.75	-2.63	-3.50	-4.38	-5.25	-6.13	-7.00	-7.88	-8.75	-9.63	-10.50	-11.38	-12.25

Where $i \neq j$

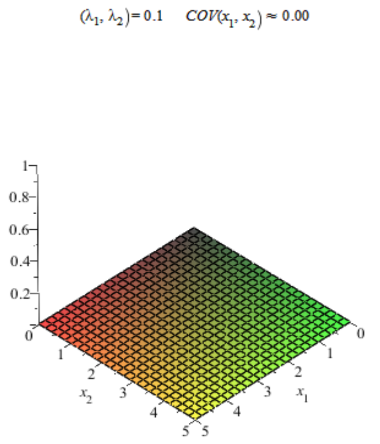

Fig 1

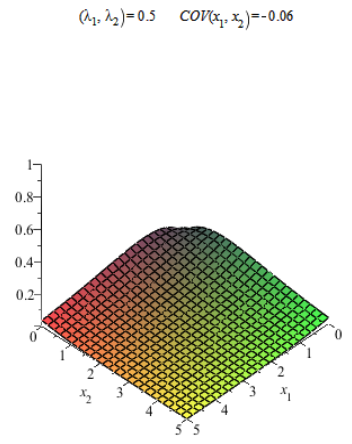

Fig 2

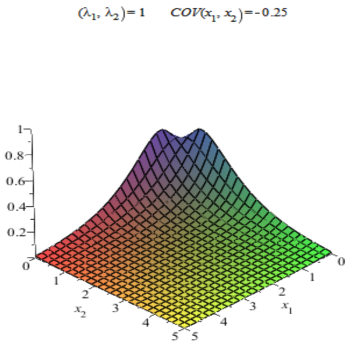

Fig 3

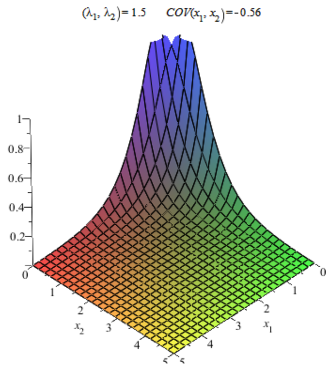

Fig 4

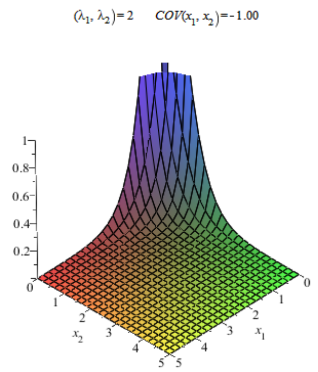

Fig 5

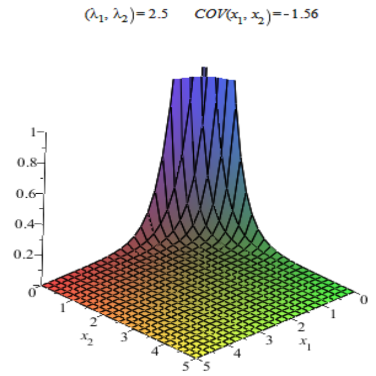

Fig6

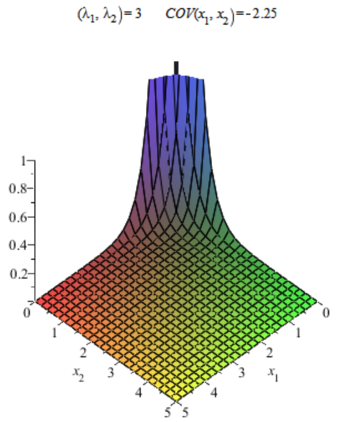

Fig 7

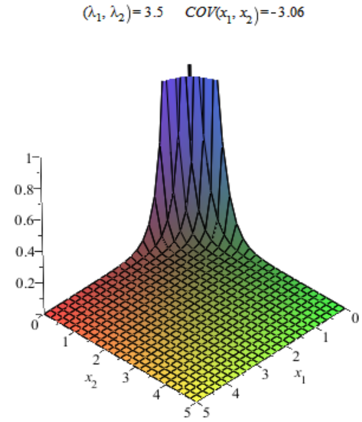

Fig 8

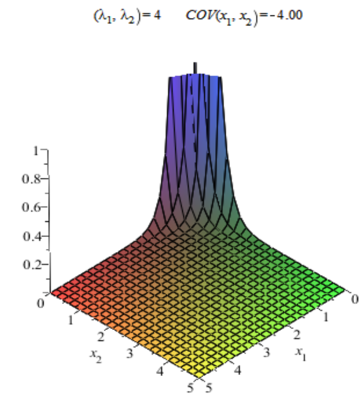

Fig 9

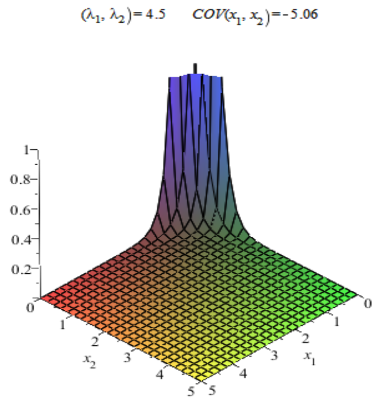

Fig 10

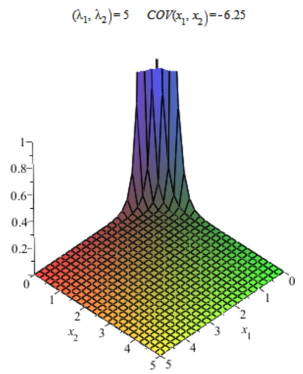

Fig 11

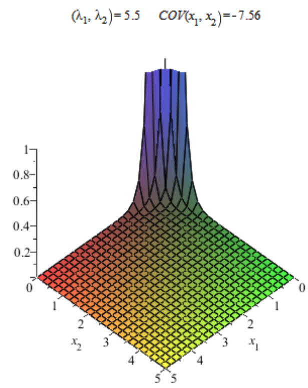

Fig 12

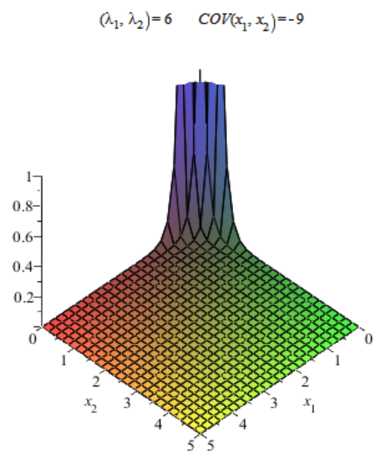

Fig 13

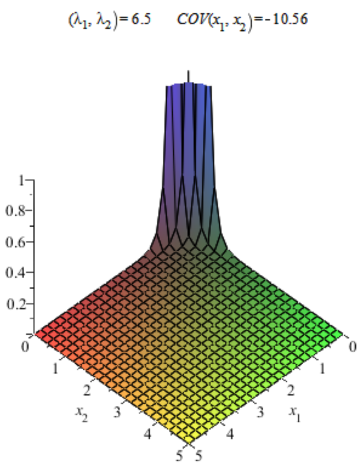

Fig 14

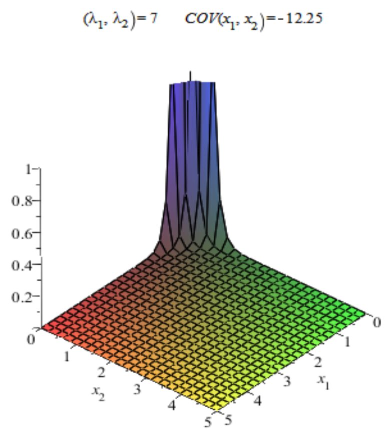

Fig 15

Result 3.5- From (1) and If $y_i = 1/x_i$, then the Sam-solai's Multivariate additive exponential distribution transformed into Sam-solai's Multivariate additive Inverse exponential distribution of Kind -2 with parameter λ_i and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(2\sum_{i=1}^{p} e^{-\frac{\lambda_i}{y_i}}) - (2^p e^{-\sum_{i=1}^{p}(\frac{\lambda_i}{y_i})}) - (p-2)\}(\prod_{i=1}^{p} \frac{\lambda_i}{y_i^2})e^{-\sum_{i=1}^{p}(\frac{\lambda_i}{y_i})} \quad (24)$$

where $0 < y_i < +\infty$, $\lambda_i > 0$

Result 3.6-From (1) and If λ_i=1and $y_i = \theta_i \sqrt[k_i]{x_i}$, the Sam-solai's Multivariate additive exponential distribution transformed into Sam-solai's Multivariate additive Weibull distribution with Parameters $(\theta_{i,} k_i)$ and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(2\sum_{i=1}^{p} e^{-(\frac{y_i}{\theta_i})^{k_i}}) - (2^p e^{-\sum_{i=1}^{p}(\frac{y_i}{\theta_i})^{k_i}}) - (p-2)\}(\prod_{i=1}^{p} (\frac{1}{\theta_i})^{k_i} k_i y_i^{k_i-1})e^{-\sum_{i=1}^{p}(\frac{y_i}{\theta_i})^{k_i}} \quad (25)$$

where $0 \le y_i < \infty$ $\theta_{i,} k_i > 0$

Result 3.7-From (1) and If $y_i = e^{-x_i} / k_i$, then the Sam-solai's Multivariate additive exponential distribution reduced into Sam-solai's Multivariate Power law distribution with parameters λ_i, k_i and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(2\sum_{i=1}^{p} (k_i y_i)^{\lambda_i}) - (2^p \prod_{i=1}^{p} (k_i y_i)^{\lambda_i}) - (p-2)\}(\prod_{i=1}^{p} \lambda_i k_i^{\lambda_i} y_i^{\lambda_i - 1}) \quad (26)$$

where $0 < y_i < \dfrac{1}{k_i}$ $\lambda_i, k_i > 0$

Result 3.8- From (1) and If $\lambda_i = 1/2$,then the Sam-solai's Multivariate additive Exponential distribution modified into Sam-solai's Multivariate additive chi-square χ^2_2 -distribution with 2 degrees of freedom and its density function is given as

$$f(x_1, x_2, x_3 \dots, x_p) = \{(2\sum_{i=1}^{p} e^{-\frac{x_i}{2}}) - (2^p e^{-\sum_{i=1}^{p}(\frac{x_i}{2})}) - (p-2)\}(\frac{1}{2})^p e^{-\sum_{i=1}^{p}(\frac{x_i}{2})} \quad (27)$$

where $0 \le x_i < \infty$

Result 3.9- From (1) and If $\lambda_i = 1/2\sigma_i^2$ and $y_i = \sqrt{x_i}$, then the Sam-solai's Multivariate additive exponential distribution transformed into Sam-solai's Multivariate additive Rayleigh distribution with parameter σ_i and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{(2\sum_{i=1}^{p} e^{-\frac{y_i^2}{2\sigma_i^2}}) - (2^p e^{-\sum_{i=1}^{p}(\frac{y_i^2}{2\sigma_i^2})}) - (p-2)\}(\prod_{i=1}^{p} \frac{y_i}{\sigma_i^2})e^{-\sum_{i=1}^{p}(\frac{y_i^2}{2\sigma_i^2})} \quad (28)$$

where $0 \le y_i < \infty$ $\sigma_i > 0$

Result 4.0- From (1)If $y_i = k_i e^{x_i}$, then the Sam-solai's Multivariate additive Exponential distribution transformed into Sam-solai's Multivariate additive Pareto distribution with parameters $(\lambda_{i,} k_i)$ and its density function is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \left\{\left(2\sum_{i=1}^{p}\left(\frac{k_i}{y_i}\right)^{\lambda_i} - 2^p \prod_{i=1}^{p}\left(\frac{k_i}{y_i}\right)^{\lambda_i}\right) - (p-2)\right\}\left(\prod_{i=1}^{p} \frac{\lambda_i k_i^{\lambda_i}}{y_i^{\lambda_i - 1}}\right) \quad (29)$$

where $k_i \le y_i < \infty$ $\lambda_i, k_i > 0$

Result 4.1-From(1)If $\lambda_i = 1$ and $y_i = \mu_i - \beta_i \log\left(\frac{e^{-x_i}}{1-e^{-x_i}}\right)$,then the Sam-solai's Multivariate additive Exponential distribution transformed into Sam-solai's Multivariate logistic distribution of Kind -2 with parameters μ_i and β_i and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=\{(2\sum_{i=1}^{p}(\frac{e^{-\frac{y_i-\mu_i}{\beta_i}}}{1+e^{-(y_i-\mu_i/\beta_i)}})-(2^p\prod_{i=1}^{p}\frac{e^{-\frac{y_i-\mu_i}{\beta_i}}}{1+e^{-(y_i-\mu_i/\beta_i)}})-(p-2)\}(\prod_{i=1}^{p}\frac{e^{-\frac{y_i-\mu_i}{\beta_i}}}{\beta_i(1+e^{-(y_i-\mu_i/\beta_i)})^2}) \tag{30}$$

where $-\infty < y_i < +\infty, -\infty < \mu_i < +\infty, \beta_i > 0$

Result 4.2- From (1) If $\lambda_i = 1$ and $y_i = \mu_i - \sigma_i \log x_i$, then the Sam-solai's Multivariate additive exponential distribution transformed into Sam-solai's Multivariate Generalized extreme value distribution of Kind -2 with parameters μ_i and σ_i and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=\{(2\sum_{i=1}^{p}e^{-e^{-(y_i-\mu_i/\sigma_i)}})-(2^p e^{-\sum_{i=1}^{p}e^{-(y_i-\mu_i/\sigma_i)}})-(p-2)\}(\prod_{i=1}^{p}\frac{e^{-\{(y_i-\mu_i/\sigma_i)+e^{-(y_i-\mu_i/\sigma_i)}\}}}{\sigma_i}) \tag{31}$$

where $-\infty < y_i < +\infty, -\infty < \mu_i < +\infty, \sigma_i > 0$

Result 4.3-From (1) If $y_i = 1 + x_i$,then the Sam-solai's Multivariate additive exponential distribution transformed into Sam-solai's Multivariate Benktander weibull distribution of Kind -2 with parameter λ_i and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=\{(2\sum_{i=1}^{p}e^{-\lambda_i(y_i-1)})-(2^p e^{-\sum_{i=1}^{p}\lambda_i(y_i-1)})-(p-2)\}(\prod_{i=1}^{p}\lambda_i)e^{-\sum_{i=1}^{p}\lambda_i(y_i-1)} \tag{32}$$

where $1 < y_i < +\infty, \lambda_i > 0$

CONCLUSION

The multivariate generalization of exponential distribution in an additive form of Sam-Solai's generalization having some interesting features. At first, the marginal univariate distributions of the Sam-Solai's Multivariate additive Exponential distribution are univariate and enjoyed the symmetric property. Secondly, the Population Correlation co-efficient of any two exponential random variables is found to be -0.25 and correlation co-efficient is independent from its Co-variance. Moreover,the authors simulated and proposed standard Co-variance matrix for different values of parameters. Finally, the multivariate generalization of exponential distribution in an additive form open the way for the same additive form of the generalization of Multivariate Inverse exponential distribution, Multivariate Weibull distribution, Multivariate Power law distribution, Multivariate chi-square distribution with two d.f, Multivariate Rayleigh distribution, Multivariate Pareto distribution, Multivariate logistic distribution, Multivariate Generalized extreme value distribution and Multivariate Benktander weibull distribution.

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