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THE APPLICATION OF THE COMPUTATION OF MATRICES IN MEASUREMENT ADJUMENT

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ABSTRACT

In this paper, we will introduce the application of the decomposition of matrices in measurement. Because of many reasons, not only the immanent causes, but also the external causes, make the results of the measurement in topography not very precise. In order to improve the precision of the measurement result, we need to measure more dates than that we need, which we call them as the redundant measurement, the consequences of these is the inconsistency of the results. then the aim of the measurement adjument is to reduce these errors. In this paper, only the condition adjustment of connecting traverse will be discussed. In order to handle the measurement dates, mathematical model should be built. In connecting traverse, there are three equations, the number of equations are equal to that of the redundant measurement. For the condition adjument, the model of it is linear, but not all measurement problems will cause the adjustment problems, it only appears when we do the redundant measurement. The method of solving this model is the least squares method, usual it is a good method to solve this problem, but when the ill-conditioned matrix arise, such as the coefficient matrix of the normal equation, the inaccuracy of the result will be increased. So a improved singular value decomposition which also combined with the method of distract the error will be introduced to decrease the inaccuracy in this paper.

Keyword: *Condition adjustment; Ill-conditioned matrix; The least square method; The truncated singular value decomposition method.*

Classification codes: *15A60.*

1. INTRODUCTION

Many scholars have already discussed this problem, such as Ye Songlin, Liu Dingyou, Xiao yesheng, Guo Luguang, Fan Gongyu，Huang Youcai and so on, they apply the ridge estimate, the matrix triangular factorization, the truncated singular value decomposition method to deal this problem. Use these methods the error of the measurement results will be reduced. In this paper, I will use a new method in the solution of the conditioned adjument, it is the connection of the truncated singular value decomposition method and the method of distracted the error.

Definition 1[1], [2]. Observation necessary elements: The elements that can only definite a geometric model. We usual denote it as t.

Definition 2[1],[2]. Redundant observation number: In the geometric sense of a model, if the total number of the observation is n, the number of the observation necessary elements is t, then we call r = n-t is the redundant observation number.

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Definition 3[1],[2]. Condition number: Let $A \in R^{n\times n}$, then we call $cond\left(A\right)=\|A\|\|A^{-1}\|$ is the condition number of matrix A.

Definition 4[1],[2]. Ill-conditioned matrix: For the condition number $cond\left(A\right)$ of matrix A, if $cond\left(A\right)>100$, then we call A is a ill-conditioned matrix.

Definition 5[1],[2]. Error compensation: The process of looking for the new optimum estimation of a set of observed value in a certain standard.

Definition 6[1],[2]. The method of condition adjustment: If there are n observed values exist, the number of the observation necessary is t, Then there are $r=n-t$ condition equations, that is $F\left(\tilde{L}\right)=0$, if the condition equation is linear, then it can be written as

$$A\tilde{L}+A_0=0 \tag{1.1}$$

where $A \in R^{r\times n}$, $\tilde{L} \in R^{n\times 1}$, $A_0 \in R^{r\times 1}$, we use $L+\Delta$ to replace $\tilde{L}$ in (1.1), and set

$$W=-(AL+A_0) \tag{1.2}$$

then (1.1) becomes

$$A\Delta - W = 0 \tag{1.3}$$

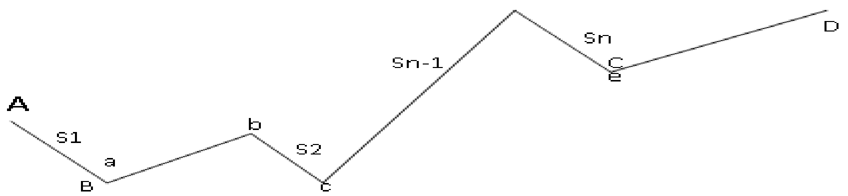
Formula (1) or (1.3) is the functional model of the condition adjustment, the computation of error compensation that based on this model is called the method of condition adjustment.

Definition 7[1],[2]. Weight of measurement: In the surveying, we call the specific gravity is the weight of measurement, it has much to do with the standard deviation of the observed value, the weight of measurement is smaller when the standard deviation is bigger, the weight of measurement of the observed value L_i is

$$p_i=\frac{\sigma_0^2}{\sigma_i^2} \tag{1.4}$$

where σ_0^2 is a arbitrary constant, which unequal to zero, σ_i^2 is the variance of L_i. If $\sigma_0^2=\sigma_1^2$, then we take the standard deviation σ_1 of L_1 as the standard.

2. THE PROCESS OF SOLVING THE CONDITION ADJUSTMENT



Be shown as in the figure, it is a unitary connecting traverse, the number of the given points is 4, the number of the unknown points is $n-1$, the number of the observed values of horizontal angle is $n+1$,the number of the observed value of the length of sides is n, while the total number of the observed value is $2n+1$. According to the analyze of the figure, in order to definite the coordinates of the unknown points, we must measure a traverse side and a horizontal angle; In order to definite all the $n-1$ unknown points, there are $n-1$ traverse sides and $n-1$ horizontal angles should be measure, then the number of the redundant observations is 3, that is to say in the unitary connecting traverse, there are three conditioned equations, they are:

(1). The conditional expression of the azimuth angle:

$$\left[v_{\beta_i}\right]_1^{n+1} - \omega_T = 0 \tag{2.1}$$

Where $\omega_T = -\left(T_0 - [\beta_i]_1^{n+1} \pm (n+1)\bullet 180^\circ - T_{CD}\right)$.

(2). The conditional expression of the vertical coordinates:

$$\left[\cos T_i \bullet v_{s_i}\right]_1^n - \frac{1}{2062.65}\left[(y_{n+1} - y_i)v_{\beta_i}\right]_1^n - \omega_x = 0 \tag{2.2}$$

Where $\omega_x = -\left(x_{n+1} - \overline{x}_C\right)$.

(3). The conditional expression of the vertical coordinates:

$$\left[\sin T_i \bullet v_{s_i}\right]_1^n - \frac{1}{2062.65}\left[(x_{n+1} - x_i)v_{\beta_i}\right]_1^n - \omega_y = 0 \tag{2.3}$$

Where $\omega_y = -\left(y_{n+1} - \overline{y}_C\right)$.

In the figure, the value of the azimuth angle of side AB is $T_{AB} = T_0$, the value of side CD is T_{CD}, whose compute value is T_{n+1}, $\left(\overline{x}_B, \overline{y}_B\right)$ is the given coordinate of B or we can note it as $\left(\overline{x}_1, \overline{y}_1\right)$, $\left(\overline{x}_C, \overline{y}_C\right)$ is the given coordinate of C and we can note it as $\left(x_{n+1}, y_{n+1}\right)$.

In the process of solving the condition adjustment of the connecting traverse, we have to compute the approximate coordinate and the approximate azimuth angle of every point, the computation formula of them are:

$$T_i = \left[\beta_j\right]_1^i + T_0 \pm i \bullet 180 \tag{2.4}$$

where T_i is the azimuth angle of the approximate coordinate of the ith side.

The adjusted value of the x-coordinate of the ith points is $\hat{x}_i$, and

$$\widehat{x_{i+1}} = x_B + \left[\Delta x_j\right]_1^i \tag{2.5}$$

$\Delta\hat{x}$ is the increase of the approximate x-coordinate, and

$$\Delta\hat{x}_i = \hat{S}_i \cos\hat{T}_i \tag{2.6}$$

The adjusted value of the vertical coordinate of the ith point is $\hat{y}_i$, and

$$\widehat{y_{i+1}} = y_B + \left[\Delta y_j\right]_1^i \tag{2.7}$$

$\Delta\hat{y}$ is the increase of the approximate vertical coordinate of the ith point, and

$$\Delta\hat{y}_i = \hat{S}_i \sin\hat{T}_i \tag{2.8}$$

The formula (2.1), (2.2), (2.3) forms the condition equations of the unitary connecting traverse.

In order to solve the equation, we write the three formulas as the form of matrix function

$$AX - W = 0 \tag{2.9}$$

Where $A \in R^{m \times n}$, $W \in R^{m \times 1}$.

the right array of this connecting traverse is

$$P = diag(P_1, P_2, \cdots, P_n) \tag{2.10}$$

Where $p_i = \dfrac{\hat{\sigma}_0^2}{\hat{\sigma}_i^2}$, σ_0^2 is a arbitrary constant that unequal to zero, σ_i^2 is the variance of L_i.

In order to use the least square method, we introduce the coefficient matrix of the lagranges interpolation,

$$K = \left[k_1, k_2, \cdots, k_n\right]^T \tag{2.11}$$

and set

$$\Phi = X^T PX - 2K^T \left(AX - W\right) \tag{2.12}$$

for formula (2.12), we do the differential coefficient of X on both sides and set it as zero, then it becomes

$$X^T P = K^T A \tag{2.13}$$

$$X = P^{-1} A^T K \tag{2.14}$$

We substitute it into (2.9), then we can obtain

$$AP^{-1} A^T K - W = 0 \tag{2.15}$$

and set $N = AP^{-1} A^T$, then

$$NK - W = 0$$

$$NK = W \tag{2.16}$$

$$K = N^{-1} W \tag{2.17}$$

then we substitute it into (2.14), then we can compute the unknown matrix X, the adjusted value can also be obtained.

3. THE IMPROVED SINGULAR VALUE DECOMPOSITION

In article [9] and [10] the scholars have already discussed the method of solving the ill-conditioned linear system of equations. In this part we will introduce a new method, and it is the combination of two methods.

For the linear system of equations

$$AX = b \tag{3.1}$$

Where $A \in R^{n \times n}$, $b \in R^{n \times 1}$.

If A is an ill-conditioned matrix, in order to decrease the error, we can not compute the inverse of the coefficient matrix directly, if we use the new method which we will introduce next the error can be decreased.

First we should compute the singular value decomposition of A,

$$A = UDV^T \tag{3.2}$$

Both U and V are unitary matrices, D is a diagonal matrix.

In order to weaken the inaccuracy, we should select a boundary value, in this paper we set $a = \|A\|_2 \bullet 10^{-2}$.

Then the truncated matrix of D becomes $D_1 = diag(\sigma_1, \sigma_2, \cdots, \sigma_r)$, and D_1 is a square matrix, the remaining elements of D are zero. except this, we put $U = [U_1, U_2]$, $V = [V_1, V_2]$, where $U_1 \in R^{n\times r}$, $V_1 \in R^{n\times r}$, then (3.1) becomes

$$U_1 D_1 V_1^T X = b \tag{3.3}$$

Next we will connect this method with the truncated singular value decomposition method and set

$$X = V_1 Y \tag{3.4}$$

where V_1 is a nonsingular matrix, then (3.3) becomes

$$U_1 D_1 V_1^T V_1 Y = U_1 D_1 Y = b \tag{3.5}$$

Then the inaccuracy of X transfer to matrix Y, that is to say weather there is inaccuracy on Y, the matrix X is always correct.

According to (3.5), we can obtain the answer of Y,

$$Y = (U_1 D_1)^{-1} b \tag{3.6}$$

We substitute it into (3.4), then the last answer is obtained

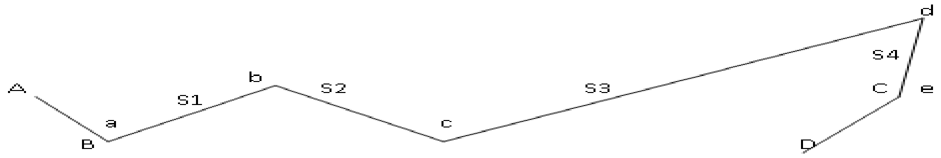
$$X = V_1 \left(U_1 D_1\right)^{-1} b$$

Using this method, we can weaken the inaccuracy of the ill-conditioned matrix, it has both the good points of the method of distract the error and the truncated singular value decomposition method, because it not only neglect the uncertain factors, but also distract the error to another matrix and we do not to seek the matrix C of the method of distract the error.

4. APPLICATION

The decomposition of matrix plays an important role in many areas, in this paper we will introduce the application of matrix in the measurement adjustment.

Example:



Be shown as in the figure, it is a unitary connecting traverse, the standard deviation of the goniometry is $\hat{\sigma}_\beta = \pm 2.5''$, the standard accuracy formula of the range finder is $\hat{\sigma}_S = 5mm + 5\times10^{-6} \bullet D$, where $5mm$ is the fixed error, 5×10^{-6} is the proportion of error coefficient, D is the scaled distance. Please solve the error compensation of this line conductor.

The known coordinates/m	The known azimuth angle
$B(187396.252,29505530.009)$ C(184817.605,29509341.482)	$T_{AB}=166^\circ 44'7.2''$ $T_{CD}=249^\circ 30'27.9''$
The observed value of the wire length/m	The observed value of the turning angle
$S_1=1478.404$ $S_2=1427.737$ $S_3=17530.352$ $S_4=1954.432$	$a=81^\circ 30'21''$ $b=252^\circ 32'32.2''$ $c=129^\circ 4'33.3''$ $d=270^\circ 20'20.2''$ $e=244^\circ 18'30''$

For the figure, the number of the unknown points is 3, the number of the traverse sides is 4, the number of the observing angles is 5, according to the $MATLAB$ we can compute the approximate coordinate and the approximate azimuth angle, they are as follows:

The approximate coordinate/m	The approximate azimuth angle
2(187944.29636,29506903.151) 3(186838.37128,29507806.194) 4(186882.19716,29525336.546) 5(184927.76516,29525329.706)	$a=68^\circ 14'28.3''$ $b=140^\circ 46'0.5''$ $c=89^\circ 51'33.8''$ $d=180^\circ 11'54''$ $e=244^\circ 30'24''$

(1). The condition equation of the correction:

The closed error of the condition equations of the correction are:

$$\omega_1 = -\left(e - T_{CD}\right) = 3.9''$$

$$\omega_2 = -\left(x_5 - x_c\right) = -1101.6$$

$\omega_3 = -(y_5 - y_c) = -159880$

The condition error of the correction is:

$$\left[\upsilon_{\beta_i}\right]_1^5 - \omega_1 = 0$$

$$\left[\cos T_i \bullet \upsilon_{s_i}\right]_1^4 - \frac{1}{2062.65}\left[(y_5 - y_i)\upsilon_{\beta_i}\right]_1^4 - \omega_2 = 0$$

$$\left[\sin T_i \bullet \upsilon_{s_i}\right]_1^4 + \frac{1}{2062.65}\left[(x_5 - x_i)\upsilon_{\beta_i}\right]_1^4 - \omega_3 = 0$$

That is

$$\upsilon_{\beta_1} + \upsilon_{\beta_2} + \upsilon_{\beta_3} + \upsilon_{\beta_4} + \upsilon_{\beta_5} - 3.9 = 0 \tag{4.1}$$

$$0.3707\upsilon_{s_1} - 0.7746\upsilon_{s_2} + 0.0025\upsilon_{s_3} - \upsilon_{s_4} - 9.5992\upsilon_{\beta_1} - 8.9334\upsilon_{\beta_2} - 8.4956\upsilon_{\beta_3} + 0.0033\upsilon_{\beta_4} + 1101.6 = 0 \tag{4.2}$$

$$0.9288\upsilon_{s_1} + 0.6325\upsilon_{s_2} + \upsilon_{s_3} - 0.0035\upsilon_{s_4} - 1.1968\upsilon_{\beta_1} - 1.4625\upsilon_{\beta_2} - 0.9263\upsilon_{\beta_3} - 0.9475\upsilon_{\beta_4} + 159880 = 0 \tag{4.3}$$

We set

$$X = [\upsilon_{s_1}, \upsilon_{s_2}, \upsilon_{s_3}, \upsilon_{s_4}, \upsilon_{\beta_1}, \upsilon_{\beta_2}, \upsilon_{\beta_3}, \upsilon_{\beta_4}, \upsilon_{\beta_5}]^T$$

The coefficient matrix is A, the constant matrix is W, then (4.1)-(4.3) becomes

$$AX - W = 0 \tag{4.4}$$

(2). Define the right of the observed value of the sides and angles

We set the standard deviation of the weight unit is $\hat{\hat{\sigma_0}} = \sigma_\beta = \pm 2.5''$, according to the standard accuracy formula we can compute the standard deviation of the trilateration.

From the introduction we can obtain the right of the observing angle, they are:

$$p_\beta = 1$$

The formula of the right of the observed value of the trilateration is

$$p_s = \frac{\hat{\sigma}_0^2}{\hat{\sigma}_s^2}$$

Then the right array of the observed value is:

$$P = diag(4.07, 4.2417, 0.0728, 2.8641, 1, 1, 1, 1, 1)$$

In order to use the least square method, we introduce the coefficient matrix

$$K = [k_1, k_2, k_3]^T$$

and set

$$\Phi = X^T PX - 2K^T(AX - W) \tag{4.5}$$

For formula (4.5), we do the differential coefficient of X on both sides and set it as zero, then

$$\frac{d\Phi}{dX} = \frac{\partial X^T PX}{\partial X} - 2\frac{\partial K^T AX}{\partial X} = 2X^T P - 2K^T A = 0 \tag{4.6}$$

By abbreviation, we can obtain

$$X = P^{-1}A^T K \tag{4.7}$$

We substitute it into (4.4),

$$AP^{-1}A^T K - W = 0 \tag{4.8}$$

This is the normal equation of (4.4), we set the coefficient matrix of (4.8) as $N = AP^{-1}A^T$ and N is a full rank matrix, By computing we can obtain

$$N = \begin{pmatrix} 5 & -27.0249 & -4.533 \\ -27.0249 & 244.6503 & 32.4235 \\ -4.533 & 32.4235 & 19.3681 \end{pmatrix}$$

Now (4.8) becomes

$$NK = W \tag{4.9}$$

For matrix N, the condition number of it is $a = cond(A) = 130.9708$, that is to say, N is an ill-condition matrix.

The singular value of it is $1.9261,\ 14.8335,\ 252.2588$, because the biggest value is much bigger than the smallest one, in order to decrease the error, we should use the new method to solve the equation. Then we will compare the results of the two methods.

(i). The method of the improved singular value decomposition

$$\mathrm{K}_1 = 10^5 \bullet [0.0592, 0.146, -1.0519]^T$$

$\mathrm{X}_1 = 10^6 \bullet [-0.0227, -0.0184, -1.443, -0.005, -0.0083, 0.0294, -0.0207, 0.1056, 0.0059]^T$ the standard deviation of the weight unit is

$$\sigma_{10} = 2.3666 \times 10^5$$

(ii). The normal method

$$N = \begin{pmatrix} 0.5116 & 0.0522 & 0.0323 \\ 0.522 & 0.0106 & -0.005 \\ 0.0323 & -0.0055 & 0.0684 \end{pmatrix}$$

$$\mathrm{K}_2 = 10^5 \bullet [-0.5221, 0.0867, -1.0929]^T$$

$$\mathrm{X}_1 = 10^6 \bullet [\text{-0.0241,-0.0179,-1.5007,-0.0029,-0.0047,0.0302,-0.0246,0.0514,-0.0522}]^T$$

The standard deviation of the weight unit is

$$\hat{\sigma}_{20} = 2.4127 \times 10^5$$

According to the two results, we can see that the new method will improve the precision of the ill-conditioned equation.

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