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A NEW PROOF FOR EUCLID'S THEOREM

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ABSTRACT
In this paper, We would like to give a new proof that there exist infinitely many primes.

***Keywords**: Prime Factors, Primes, Euclid's Theorem.*

1 INTRODUCTION

Theorem 1: (Fundamental Theorem of Arithmetic) Every natural number is a product of primes. [3]

Theorem 2: (Euclid's Theorem) There are infinitely many primes. [3]

The title of this section is surely, along with the uniqueness of factorization, the most basic and important fact in number theory. Euclid may have been the first to give a proof that there are infinitely many primes. There are several different proofs for Euclid's Theorem with many variants, and some of them can be found in [1, 2, 4, 5, 6, 7]. I would like to add such proofs later.

Stieltjes' proof:
Assume that $\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_r}$ are the only primes. Let N = $\mathbf{p_1 p_2 \ldots p_r}$ and let N = mn be any factorization of N with m, n ≥1. Since each prime $\mathbf{p_i}$divides exactly one of m and n, none of $\mathbf{p_i}$'s divides m + n. This means m + n is divisible by none of the existing primes, which is impossible since m + n > 1. [7]

Kummer's proof:
Suppose that there exist only finitely many primes $\mathbf{p}_1 < \mathbf{p}_2 < \ldots < \mathbf{p}_r$. Let $\mathbf{N} = \mathbf{p}_1 \cdot \mathbf{p}_2 \cdot \ldots \cdot \mathbf{p}_r$. The integer **N-**1, being a product of primes, has a prime divisor $\mathbf{p}_i$ in common with **N**; so, $\mathbf{p}_i$ divides **N - (N-**1) =1, which is absurd. [7]

Filip Saidak's Proof:
Let n > 1 be a positive integer. Since n and n+1 are consecutive integers, they must be coprime, and hence the number N_2 = n(n + 1) must have at least two different prime factors. Similarly, since the integers n(n+1) and n(n+1)+1 are consecutive, and therefore coprime, the number N_3 = n(n + 1)[n(n + 1) + 1] must have at least 3 different prime factors. This can be continued indefinitely. [6]

Braun's proof:
Suppose that there exist only t primes $\mathbf{p_1}, \mathbf{p_2}, \ldots, \mathbf{p_t}$ and consider the sum m/n $=\sum_{i=1}^{t} 1/p_i$.Now1/2+1/3+1/5 > 1, so that m/n > 1. Therefore, m > n ≥1. Thus m must have a prime factor $\mathbf{p_i}$. However, no $\mathbf{p_i}$ can divide m since $\mathbf{p_i} \mid$ m implies $\mathbf{p_i} \mid \mathbf{p_1 p_2 \ldots p_{i-1} p_{i+1} \ldots p_t}$. This is a contradiction. [7]

2. A NEW PROOF FOR EUCLID'S THEOREM

We know that if there exists an infinite sequence of primes that all the members are prime together, then there are infinitely many primes [6].

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In this paper, Euclid's Theorem proved by introducing an infinite sequence which all the members are prime together.

In this case, few sequences are introduced that satisfies in the above condition. For example, Mersenne's Sequence $\{M_p = 2^p-1\}$or Fermat's Sequence $\{F_n = 2^{2^n}+1\}$. [6]

Now, We define a infinitely sequence such that satisfies in the above condition entitled khosravi's sequence.

Theorem 3: (Khosravi's Sequence) There are infinitely many primes.

Proof: We define a infinitely sequence A as follows,

$A = \{a_n\} = \{n, n!-1, n!!-1, n!!!-1, n!!!!-1 \ldots\}$.

Clearly, all members of A are prime together (for $n > 2$). Therefore, the proof is completed.

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